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RESEARCH ARTICLE

ON $\hat{\hat{g}}$ -CLOSED SETS IN BITOPOLOGICAL SPACES

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A B S T R A C T

In this paper we introduce $\hat{\hat{g}}$ -closed sets [12] in bitopological spaces. Properties of these sets are investigated and we introduce two new bitopological spaces (i, j)- $\hat{\hat{T}}_{1/2}$ spaces and (i, j)- $\hat{T}_f$ spaces as applications. Further we introduce and study $\hat{\hat{g}}$ -continuous maps [12] and $\hat{\hat{g}}$ -irresolute maps [12] in bitopological spaces.

INTRODUCTION

A triple (X, τ_1, τ_2) where X is non empty set and τ_1 and τ_2 are topologies on X is called a bitopological space. Kelly [3] initiated the study of such spaces in 1963. Fukutaka [6] introduced the concept of g-closed sets [2] in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Recently Manoj *et al* [12] introduced and studied the concepts of $\hat{\hat{g}}$ -closed sets and $\hat{\hat{g}}$ -continuity in topological spaces.

In the present paper we introduce the concept of $\hat{\hat{g}}$ -closed sets [12] in bitopological spaces and then investigate some of their properties. We also define and study new types of spaces namely $\hat{\hat{T}}_{1/2}$ -spaces [12] and $\hat{T}_f$ -spaces [12] in bitopological spaces. We further introduce new class of maps called $\hat{\hat{g}}$ -continuous maps and $\hat{\hat{g}}$ -irresolute maps in bitopological spaces and investigate their properties.

Preliminaries

If A is a subset of X with topology τ , then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c.

Definition 2.01: (i) A subset A of a topological space (X, τ) is called semi-open [1] (resp. regular open [8], pre-open [2]) if A $\subseteq$ cl(int(A)) (resp. A=int(cl(A)), A $\subseteq$ int(cl(A))). (ii) A subset A of a topological space (X, τ) is called a generalized closed [2] (resp. sg-closed [15], $\hat{\hat{g}}$ -closed [12]) set if cl(A) $\subseteq$ U (resp. scl(A) $\subseteq$ U, cl(A) $\subseteq$ U) whenever A $\subseteq$ U and U is open (resp. semi-open, sg-open) in X.

Definition 2.02: The intersection of all pre closed sets containing A is called the pre closure of A and denoted by τ-pcl(A) or pcl(A).

Throughout this paper X and Y always represent non-empty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise explicitly mentioned and integers i, j, k $\in$ {1, 2}. For a subset A of X, τ_i -cl(A) (resp. τ_i -int(A), τ_ι–pcl(A)) denote the closure (resp. interior , pre closure) of A w.r.t. topology τ_i. We denote the family of all semi generalized open (briefly sg-open) subsets of X w.r.t. the topology τ_i by SGO(X, τ_i) and the family of all τ_j-closed sets is denoted by F_j. By (i, j) we mean the pair of topologies (τ_I, τ_j).

Definition 2.03: A subset A of a bitopological space (X, τ_1, τ_2) is called :

(i) (i, j)-g-closed [6] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in$ τ_i.
(ii) (i, j)-rg-closed [4] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_ι
(iii) (i, j)-gpr-closed [13] if τ_j-pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_ι
(iv) (i, j)wg-closed [10] if τ_j-cl(τ_ι-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U $\in$ τ_i.
(v) (i, j)-ω-closed [13] if τ_j-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in τ_i.

(vi) (i, j)-g*-closed [16] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is generalized open in τ_i.

(vii) (i, j)-*g-closed [17] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$ –open in τ_i.

The family of all (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed, (i, j)-g*-closed, (i, j)-*g-closed) subsets of a bitopological space (X, τ_1, τ_2) is denoted by D(i, j) (resp. D_r(i, j), ζ(i, j), W(i, j), C(i, j), D*(i, j) and *D(i, j)).

Definition 2.04 : (i) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-$T_{1/2}$ [6] (resp. (i, j)-$T^*_{1/2}$ [16], (i, j)-$^*T_{1/2}$ [16]) if every (i, j)-g-closed (resp. (i, j)-g*-closed, (i, j)-g-closed) set is τ_j–closed (resp. τ_j–closed, (i, j)-g*–closed).

A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise $T_{1/2}$ [6] (resp. strongly pairwise $T^*_{1/2}$ [16]) space if it is (1, 2)-$T_{1/2}$ and (2, 1)-$T_{1/2}$ (resp. (1, 2)-$T^*_{1/2}$ and (2, 1)-$T^*_{1/2}$) space.

Definition 2.05 : A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

(i) τ_j-σ_k-continuous [5] if $f^{-1}(V) \in \tau_j$ for every V ∈ σ_k.

(ii) D (i, j)-σ_k-continuous [5] (resp. D_r (i, j)-σ_k-continuous [4], ζ(i, j)-σ_k-continuous [13] W (i, j)-σ_k-continuous [10], C (i, j)-σ_k-continuous [13], D* (i, j)-σ_k-continuous [16] and *D (i, j)-σ_k-continuous [17]) if the inverse image of every σ_k-closed set is (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed, (i, j)-g*-closed and (i, j)-*g-closed) set in (X, τ_1, τ_2).

Definition 2.06: A topological space (X, τ) is called $\hat{\hat{T}}_{1/2}$ -space [12] (resp. $\hat{T}_f$ -space [12]) if every $\hat{\hat{g}}$ -closed set (resp. g-closed set) is closed (resp. $\hat{\hat{g}}$ -closed).

(i, j) - $\hat{\hat{g}}$ –Closed Sets

In this section we introduce the concepts of (i, j)- $\hat{\hat{g}}$ -closed sets in bitopological spaces.

Definition 3.01: A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $\hat{\hat{g}}$ -closed set if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\subseteq$ SGO (X, τ_i).

We denote the family of all (i, j)- $\hat{\hat{g}}$ -closed sets in (X, τ_1, τ_2) by $\hat{\hat{D}}$(i, j).

Remark 3.02: By setting $\tau_1 = \tau_2$ in definition (3.01), an (i, j)- $\hat{\hat{g}}$ -closed set is $\hat{\hat{g}}$ -closed set.

Proposition 3.03: If A is τ_j-closed subset of (X, τ_1, τ_2) , then A is (i, j)- $\hat{\hat{g}}$ -closed.

The converse of the above proposition is not true as seen from the following example.

Example 3.04: X = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X} and τ_2 = {ϕ, {a, b}, X} then the subset {a, b} is (1, 2)- $\hat{\hat{g}}$ -closed but not τ_2-closed.

Proposition 3.05 : In a bitopological space every an (i, j)- $\hat{\hat{g}}$ -closed set is (i) (i, j)-g-closed set (ii) (i, j)-rg-closed set (iii) (i, j)-gpr-closed set (iv) (i, j)-ω-closed set (v) (i, j)-Wg-closed (vi) (i, j)-*g-closed set.

The following examples show that the reverse implications of the above proposition are not true.

Example 3.06: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b, c}, X} and τ_2 = {ϕ, {a}, {a, b}, X}. Then the subset {a, b} is (1, 2)-g-closed but not (1, 2)- $\hat{\hat{g}}$ -closed.

Example 3.07: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {c}, {a, c}, X} and τ_2 = {ϕ, {a}, X}. Then the subset {a, b} is (1, 2)-rg-closed but not (1, 2)- $\hat{\hat{g}}$ -closed.

Example 3.08: In example (3.04), the subset {b, c} is (1, 2)-gpr-closed but not (1, 2)- $\hat{\hat{g}}$ -closed.

Example 3.09: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ, {a}, {b, c}, X}. Then the subset {a, b} is (2, 1)-ω-closed but not (2, 1)- $\hat{\hat{g}}$ -closed.

Example 310: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {b}, X}. Then the subset {a, b} is (1, 2)-Wg-closed but not (1, 2)- $\hat{\hat{g}}$ -closed.

Example 3.11: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ, {c}, {a, c}, X}. Then the subset {c} is (1, 2)-*g-closed but not (1, 2)- $\hat{\hat{g}}$ -closed.

Remark 3.12: The following examples show that (i, j)- $\hat{\hat{g}}$ -closed set and τ_j-g-closed set are independent.

Example 3.13: In example (3.07), the subset {b} is not (1, 2)- $\hat{\hat{g}}$ -closed but it is τ_2-g-closed.

Example 3.14: In example (3.04), the subset {b, c} is (2, 1)- $\hat{\hat{g}}$ -closed but not τ_1-g-closed.

Remark 3.15: The following examples show that an (i, j)- $\hat{\hat{g}}$ -closed set and (i, j)-g*-closed sets are independent as it can be seen from the following examples.

Example 3.16: In example (3.06), the subset {a, c} is (2, 1)-g*-closed but not (2, 1)- $\hat{\hat{g}}$ -closed.

Example 3.17: In example (3.11), the subset {c} is (1, 2)- $\hat{\hat{g}}$ -closed but not (1, 2)-g*-closed.

The following diagram summarizes the above discussions.

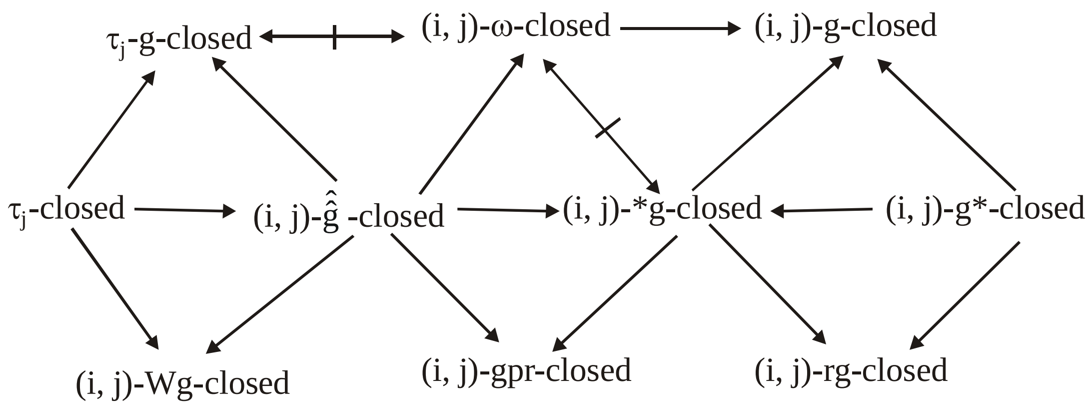

Diagram (3.18)

Where A $\rightarrow$ B (resp. A $\leftrightarrow\!\!\!\!\!\!\!\!\mid\;\;$ B) represents A implies B but not conversely (resp. A and B are independent).

Proposition 3.19: If A, B $\in \hat{D}$ (i, j), then A $\cup$ B is not necessarily belongs to $\hat{\hat{D}}$ (i, j).

Proof: Since union of two sg-open sets is not necessarily sg-open so proof is obvious.

Remark 3.20: The intersection of two (i, j)-$\hat{\hat{g}}$ -closed sets need not be (i, j)-$\hat{\hat{g}}$ -closed set as seen from the following example.

Example 3.21: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b, c}, X} and τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X} then {a, b} and {b, c} are (2, 1)-$\hat{\hat{g}}$ -closed sets but their intersection {b} is (2, 1)-$\hat{\hat{g}}$ -closed set.

Remark 3.22: In a bitopological space (X, τ_1, τ_2) , $\hat{\hat{D}}$ (1, 2) is generally not equal to $\hat{\hat{D}}$ (2, 1) as it can be seen from the following example.

Example 3.23: In example (3.09), {a} is (1, 2)-$\hat{\hat{g}}$ -closed but not (2, 1)-$\hat{\hat{g}}$ -closed and {c} is (2, 1)-$\hat{\hat{g}}$ -closed but not (1, 2)-$\hat{\hat{g}}$ -closed.

Proposition 3.24: If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) , then $\hat{\hat{D}}$ (2, 1) $\subseteq$ $\hat{\hat{D}}$ (1, 2).

The converse of the above proposition is not true as it can be seen from the following example.

Example 3.25: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_1 = {ϕ, {b}, {a, c}, X}. Then $\hat{\hat{D}}$ (2, 1) $\subseteq$ $\hat{\hat{D}}$ (1, 2) but τ_1 is not contained in τ_2.

Proposition 3.26: For each element x of (X, τ_1, τ_2), {x} is τ_i–sg-closed or $\{x\}^c$ is (i, j)-$\hat{\hat{g}}$ -closed.

Proposition 3.27: If A is (i, j)-$\hat{\hat{g}}$ -closed then τ_j-cl(A)-A contains no non-empty τ_i-sg-closed set.

The converse of the above proposition is not true as it can be seen from the following example.

Example 3.28: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X} and τ_2 = {ϕ, , {a, b}, X}. If A={c}, then τ_2-cl(A)-A = ϕ does not contain any non-empty τ_1-sg-closed set but A is not a (1, 2)-$\hat{\hat{g}}$ -closed.

Proposition 3.29: If A is (i, j)-$\hat{\hat{g}}$ -closed set in (X, τ_1, τ_2) then A is τ_j-closed iff τ_j-cl(A) is τ_i-sg-closed.

Proof: Let A is τ_j-closed then τ_j-cl(A) = A i.e. τ_2-cl(A)-A = ϕ and hence τ_j-cl(A)–A is τ_i-sg-closed.
Conversely, let τ_j-cl(A)–Ais τ_i-sg-closed, then by prop.(3.27) . τ_j-cl(A)-A = ϕ , since A is (i, j)-$\hat{\hat{g}}$ -closed so A is τ_j-closed.

Proposition 3.30: If A is an (i, j)-$\hat{\hat{g}}$-closed set of (X, τ_1, τ_2) such that A $\subseteq$ B $\subseteq$ τ_j-cl(A) then B is also an (i, j)-$\hat{\hat{g}}$ -closed set of (X, τ_1, τ_2).

Proposition 3.31: Let A $\subseteq$ Y $\subseteq$ X and A is (i, j)-$\hat{\hat{g}}$ -closed in X, then A is (i, j)-$\hat{\hat{g}}$ -closed relative to Y.

Theorem 3.32: In a bitopological space (X, τ_1, τ_2) , SGO(X, τ_I) $\subseteq$ F_j (Family of all closed sets in τ_j) iff every subset of X is an (i, j)-$\hat{\hat{g}}$ -closed set.

Proof: Let SGO(X, τ_I) $\subseteq$ F_j . Let A be a subset of X such that A $\subseteq$ U where U $\in$ SGO(X, τ_I). Then τ_j-cl(A) $\subseteq$ τ_j –cl(U) = U and hence A is (i, j)-$\hat{\hat{g}}$ -closed.

Conversely let every subset of X is (i, j)-$\hat{\hat{g}}$ -closed. Let U $\in$ SGO(X, τ_I). Since U is (i, j)-$\hat{\hat{g}}$ -closed, we have τ_j –cl(U) $\subseteq$ U. So U $\in$ F_j and hence SGO(X, τ_I) $\subseteq$ F_j .

(i, j)-$\hat{\hat{T}}_{1/2}$ -Spaces and (i, j)-$\hat{T}_f$ -Spaces

In this section we introduce the following definitions.

Definition 4.01: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $\hat{\hat{T}}_{1/2}$ space if every (i, j)- $\hat{\hat{g}}$-closed set is τ_j–closed.

Definition 4.02: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $\hat{T}_f$ space if every (i, j)- g-closed set is (i, j)- $\hat{\hat{g}}$-closed.

Definition 4.03: A bitopological space (X, τ_1, τ_2) is said to be an strongly pairwise- $\hat{\hat{T}}_{1/2}$ space if it is (1, 2)- $\hat{\hat{T}}_{1/2}$ space and (2, 1)- $\hat{\hat{T}}_{1/2}$ space.

Proposition 4.04: If (X, τ_1, τ_2) is (i, j)-$T_{1/2}$ space then it is an (i, j)- $\hat{\hat{T}}_{1/2}$ space but not conversely.

Example 4.05: In example (3.09), (X, τ_1, τ_2) is (1, 2)– $\hat{\hat{T}}_{1/2}$ space but not (1, 2)-$T_{1/2}$ space.

Proposition 4.06: If (X, τ_1, τ_2) is strongly pairwise $T_{1/2}$-space then it is strongly pairwise $\hat{\hat{T}}_{1/2}$ -space but not conversely.

Example 4.07: In example (3.07), (X, τ_1, τ_2) is strongly pairwise $\hat{\hat{T}}_{1/2}$ -space but not strongly pairwise $T_{1/2}$-space.

Theorem 4.08: A bitopological space (X, τ_1, τ_2) is an (i, j)- $\hat{\hat{T}}_{1/2}$ space iff {x} is τ_j-open or τ_i -sg-closed for each $x \in X$.

Proof: Suppose that {x] is not τ_i-sg-closed then $\{x\}^c$ is (i, j)- $\hat{\hat{g}}$ -closed by proposition (3.26). Since (X, τ_1, τ_2) is an (i, j)- $\hat{\hat{T}}_{1/2}$ space, $\{x\}^c$ is τ_j-closed i.e. $\{x\}^c$ is τ_j-open.

Conversely let F be an (i, j)- $\hat{\hat{g}}$ -closed set. By assumption {x} is τ_j-open or τ_i-sg-closed for any $x \in \tau_j$-cl(F).

Case I- Suppose {x} is τ_j-open. Since $\{x\} \cap F \neq \phi$ we have $x \in F$.

Case II- Suppose {x} is τ_j-sg-closed. If $x \notin F$ then $\{x\} \subseteq \tau_j$-cl(F)–F, which is a contradiction to proposition (3.29) so $x \in F$.

Thus in both cases we find that F is τ_j–closed i.e. (X, τ_1, τ_2) is (i, j)- $\hat{\hat{T}}_{1/2}$ space.

Remark 4.09: (i, j)- $\hat{\hat{T}}_{1/2}$ space and (i, j)- $\hat{T}_f$ space are independent to each other as seen from the following examples..

Example 4.10: In example (3.07), (X, τ_1, τ_2) is (2, 1)- $\hat{\hat{T}}_{1/2}$ space but not (2, 1)- $\hat{T}_f$ space.

Example 4.11: In example (3.04), (X, τ_1, τ_2) is (1, 2)- $\hat{T}_f$ space but not (1, 2)- $\hat{\hat{T}}_{1/2}$ space.

Proposition 4.12: If (X, τ_1, τ_2) is (i, j)-$T_{1/2}$ space then it is (i, j)- $\hat{T}_f$ space but not conversely.

Example 4.13: In example (3.04), (X, τ_1, τ_2) is (1, 2)- $\hat{T}_f$ space but not (1, 2)-$T_{1/2}$ space.

Proposition 4.14: A bitopological space (X, τ_1, τ_2) is (i, j)-$T_{1/2}$ space iff it is both (i, j)- $\hat{T}_f$ space and (i, j)- $\hat{\hat{T}}_{1/2}$ space.

Remark 4.15: In (X, τ_1, τ_2), (i, j)-$^*T_{1/2}$ space is not necessarily (i, j)- $\hat{T}_f$ space as it can be seen from the following example.

Example 4.16: In example (3.09), (X, τ_1, τ_2) is (1, 2)-$^*T_{1/2}$ space but not (1, 2)- $\hat{T}_f$ space.

Proposition 4.17: If (X, τ_1, τ_2) is (i, j)-$T^*_{1/2}$ space then it is (i, j)- $\hat{\hat{T}}_{1/2}$ space but not conversely.

Example 4.18: In example (3.07), (X, τ_1, τ_2) is (1, 2)- $\hat{\hat{T}}_{1/2}$ space but not (1, 2)-$T^*_{1/2}$ space.

Proposition 4.19: If (X, τ_1, τ_2) is strongly pairwise $T^*_{1/2}$-space then it is strongly pairwise $\hat{\hat{T}}_{1/2}$ -space but not conversely.

Example 4.20: In example (3.07), (X, τ_1, τ_2) is strongly pairwise $\hat{\hat{T}}_{1/2}$ -space but not strongly pairwise $T^*_{1/2}$-space.

The following diagram summarizes the above discussions.

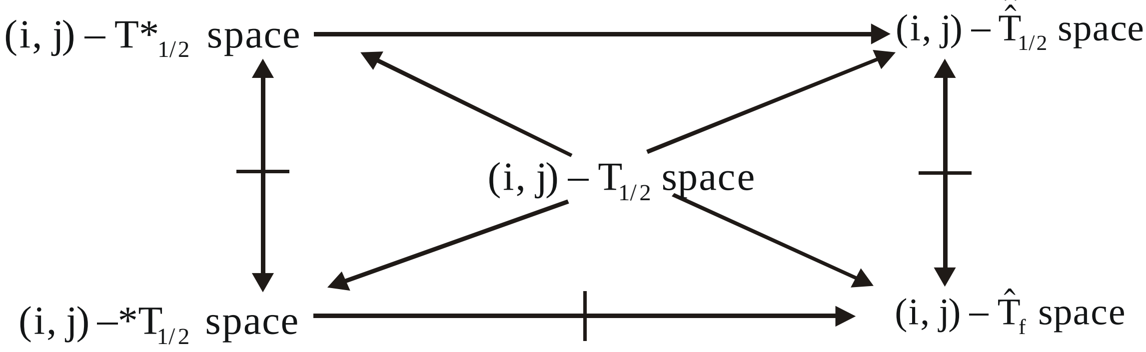

Diagram (4.21)

$\hat{\hat{g}}$ -Continuous Maps

In this section we introduce $\hat{\hat{g}}$ –continuous maps in bitopological spaces.

Definition 5.01: A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $\hat{\hat{D}}$(i, j)-σ_k-continuous if the inverse image of every σ_k -closed set is an (i, j)- $\hat{\hat{g}}$ -closed set.

Proposition 5.02: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is τ_j-σ_k-continuous then it is $\hat{\hat{D}}$(i, j)-σ_k-continuous but not conversely.

Example 5.03: X = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b c}, {a, c}, X}, τ_2 = {ϕ, {a, b}, X} and Y = {p, q}, σ_1= {ϕ, {p}, Y}, σ_2= {ϕ, {q}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = q, f(b) = p, f(c) = p. Then map f is (2, 1)- $\hat{\hat{g}}$ - σ_2-continuous but not τ_1-σ_2-continuous.

Proposition 5.04 : If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $\hat{\hat{D}}$(i, j) σ_k-continuous then it is (i) D(i, j)-σ_k-continuous (ii) D_r(i, j)-σ_k-continuous (iii) ζ(i, j)-σ_k-continuous (iv) C (i, j)-σ_k-continuous (v) W(i, j)-σ_k-continuous (vi) *D (i, j)-σ_k-continuous.

However the reverse implications of the above proposition are not true in general as it can be seen from the following examples.

Example 5.05: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {c}, {a, c}, X}, τ_2 = {ϕ, {b}, {a, b}, {b, c}, X} and Y = {p, q}, σ_1= {ϕ, {p}, Y}, σ_2= {ϕ, {q}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = p, f(b) = p, f(c) = q. Then map f is D (1, 2)-σ_2-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_2-continuous.

Example 5.06 *:* Let X = {a, b, c} = Y, τ_1 = {ϕ, {a}, {a, c}, X}, τ_2 = {ϕ}, {a, b}, X} and σ_1= {ϕ, {b}, {b, c}, Y}, σ_2= {ϕ, {a}, {a, b}, {a, c}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = b, f(b) = c, f(c) = a. Then map f is D_r (1, 2)-σ_2-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_2-continuous.

Example 5.07: Let X = {a, b, c} = Y, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = {ϕ}, {a, b}, X} and σ_1= {ϕ, {a}, {a, b}, Y}, σ_2= {ϕ, {a}, {b}, {a, b}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = b, f(b) = c, f(c) = a. Then map f is ζ (1, 2)-σ_1-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_1-continuous.

Example 5.08: Let X = {a, b, c} = Y, τ_1 = {ϕ, {a}, {b, c}, X}, τ_2 = {ϕ}, {a}, {a, b}, X} and σ_1= {ϕ, {a}, {b}, {a, b}, Y}, σ_2= {ϕ, {a}, {a, b}, {a, c}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = a, f(b) = c, f(c) = b. Then map f is C (1, 2)-σ_2-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_2-continuous.

Example 5.09: Let X = {a, b, c} = Y, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = {ϕ}, {a, b}, X} and σ_1= {ϕ,, {b}, {b, c}, Y}, σ_2= {ϕ, {c}, {b, c}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by identity mapping. Then map f is W (1, 2)-σ_2-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_2-continuous.

Example 5.10: Let X = {a, b, c} = Y, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ}, {c}, {a, c}, X} and σ_1= {ϕ, {c}, {b, c}, Y}, σ_2= {ϕ, {a}, {b}, {a, b}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = c, f(b) = a, f(c) = b. Then map f is *D(1, 2)-σ_1-continuous but not $\hat{\hat{D}}$ (1, 2)-σ_1-continuous.

Remark 5.11: $\hat{\hat{D}}$ (i, j)- σ_k-continuous and D* (i, j)-σ_k-continuous are independent as it can be seen from the following examples.

Example 5.12: Let X = {a, b, c} = Y, τ_1 = {ϕ, {a}, {b, c}, X}, τ_2 = {ϕ}, {a}, {a, b}, X} and σ_1= {ϕ, {a}, {b}, {a, b}, Y}, σ_2= {ϕ, {c}, {a, c}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by f(a) = a, f(b) = c, f(c) = b. Then map f is D* (2, 1)-σ_2-continuous but not $\hat{\hat{D}}$ (2, 1)-σ_2-continuous.

Example 5.13: Let X = {a, b, c} = Y, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ}, {b}, X} and σ_1= {ϕ, {b}, {c}, {b, c}, Y}, σ_2= {ϕ, {a}, {a, b}, Y}. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by identity mapping. Then map f is $\hat{\hat{D}}$ (1, 2)-σ_2-continuous but not D* (1, 2)-σ_2-continuous.

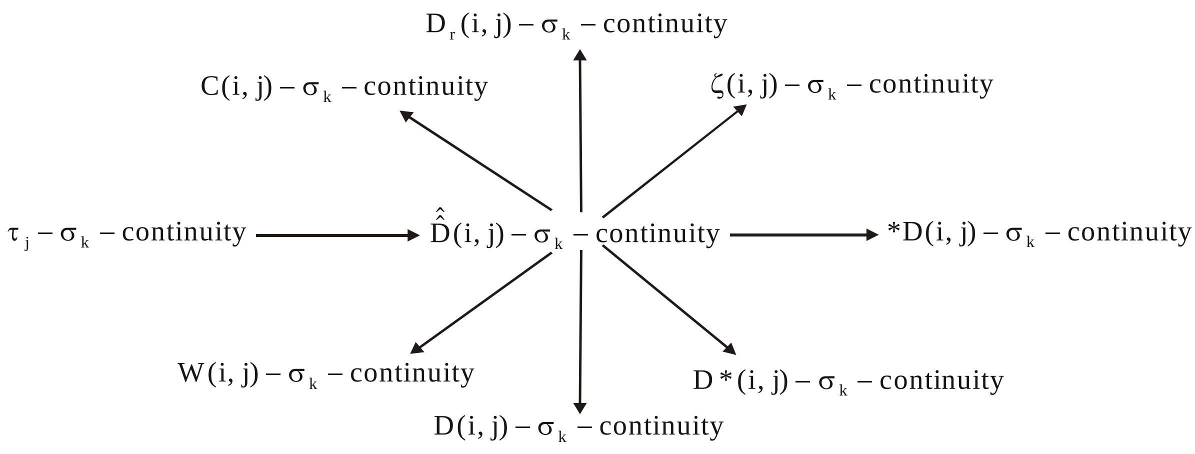

Diagram (5.18)

Definition 5.14: A map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $\hat{\hat{D}}$ (i, j)- σ_k-irresolute if the inverse image of every σ_k- $\hat{\hat{g}}$ -closed set in Y is (i, j)- $\hat{\hat{g}}$ -closed in X.

Proposition 5.15: If f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\hat{\hat{D}}$ (i, j)- σ_k-irresolute then it is $\hat{\hat{D}}$ (i, j)- σ_k-continuous but not conversely.

Example 5.16: Let X = {a, b, c} = Y, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi\}, \{a\}, \{b, c\}, X\}$ and $\sigma_1 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, Y\}$, $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a) = c, f(b) = b, f(c) = a. Then map f is (1, 2)- $\hat{\hat{g}}$ -σ_2-continuous but not (1, 2)- $\hat{\hat{g}}$ -σ_2-irresolute.

Proposition 5.17: If f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\hat{\hat{D}}$ (i, j)- σ_k-continuous and (Y, σ_k) is $\hat{\hat{T}}_{1/2}$ -space then f is $\hat{\hat{D}}$ (i, j)- σ_k-irresolute.

Proof: Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)- $\hat{\hat{g}}$ -σ_k – continuous then inverse image of every σ_k-closed set in Y is (i, j)- $\hat{\hat{g}}$ -closed in X. Since (Y, σ_k) is $\hat{\hat{T}}_{1/2}$ -space so every σ_k- $\hat{\hat{g}}$ -closed set is σ_k-closed i.e. inverse image of every σ_k- $\hat{\hat{g}}$ -closed set is (i, j)- $\hat{\hat{g}}$ -closed so map f is (i, j)- $\hat{\hat{g}}$ - σ_k-irresolute.

The following diagram summarizes the above discussions.

Theorem 5.19: Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map.

(i) If (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space then f is D(i, j)- σ_k-continuous iff it is $\hat{\hat{D}}$ (i, j)- σ_k-continuous.

(ii) If (X, τ_1, τ_2) is an (i, j)- $\hat{\hat{T}}_{1/2}$ space then f is τ_j-σ_k-continuous iff it is $\hat{\hat{D}}$ (i, j)-σ_k-continuous.

Proof: (i) Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is D (i, j)-σ_k – continuous then inverse image of every σ_k-closed set is (i, j)-g-closed in X. Since (X, τ_1, τ_2) is (i, j)-$T_{1/2}$ so every (i, j)-g-closed set is τ_j –closed but every τ_j –closed set is (i, j)- $\hat{\hat{g}}$ -closed so inverse image of every σ_k -closed set is (i, j)- $\hat{\hat{g}}$ -closed i.e. map f is $\hat{\hat{D}}$ (i, j)-σ_k-continuous.

Conversely let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\hat{\hat{D}}$ (i, j)- σ_k – continuous then by proposition (5.04) map f is D(i, j)-σ_k-continuous.
Follows as above.

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