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Research Article

MORLEY'S THEOREMOUTER TRISEKTOR ON TRIANGLES AND ISOSCELES TRAPEZOID

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ABSTRACT

Morley's theorem is applied inner angles on triangles and squares, in this paper will be developed Morley's Theorem Using the outer angle on triangles and squares (rhombus, kite and trapezoid foot). In any triangle, the Morlay theorem using an outer angle produces an equilateral triangle and the isosceles trapezoid produces an the kite. The proof in this paper uses a simple method with the concept of kekongruenanan and trigonometry concepts.

INTRODUCTION

One of the theorems about triangles that can be used as an example is the morley theorem. N.wall (2008: 12), states that Morley's theorem is the most interesting and surprising result in the 20th century in the field of geometry. Morley states that there are any triangles in which each angle is formed by a trisector, then two adjacent trisectors will intersect and if the intersection points are connected, an equilateral triangle will be formed. Bramutu, F.A. (2018: 43), states that there are rectangles in which each angle is formed in the inner trisektor, then two adjacent trisektors will intersect and if the intersection points are connected a special quadrilateral will be formed.

This paper will discuss the application and development of Morley's theorem to any triangle and quadrilateral that applies to rhombus, kite and isosceles trapezoid using a outer trisektor. The idea of proof is to use the concept of congruence and trigonometry discussed by Mashadi (a) (2015: 186) and Mashadi (b) (2016:185).

Morley's Theorem in the triangle

Any known ΔABC, the adjacent inner trisektor will intersect and if the intersection is connected it will form an equilateral triangle which can be seen in (Figure 1) .

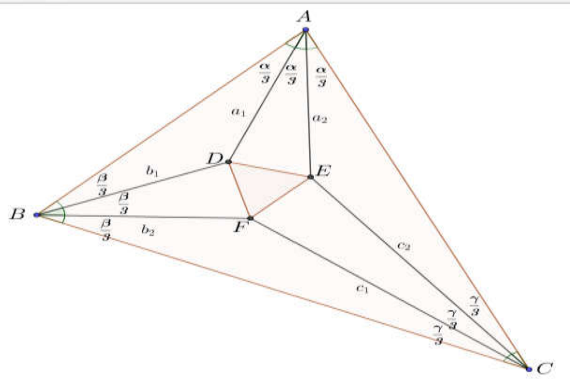

Figure 1 Morley's Theorem in the triangleΔABC.

Some evidence of Morley's theorem with a different point of view has been found by mathematicians stated that the length of the triangular side of Morley with $\frac{a}{\sin\frac{\angle\alpha}{3}} = \frac{b}{\sin\frac{\angle\beta}{3}} = \frac{c}{\sin\frac{\angle\gamma}{3}} = 2R$, adalah $8R\, sin\frac{\angle\alpha}{3} sin\frac{\angle\beta}{3} sin\frac{\angle\gamma}{3}$.

The development of the morley outher angles

Morley's Theorem on Rhombus

Rhombus■ABCDis known, the adjacent inner trisektor will intersect and if the intersection point is connected it will form a rectangle that can be seen in (Figure 2).

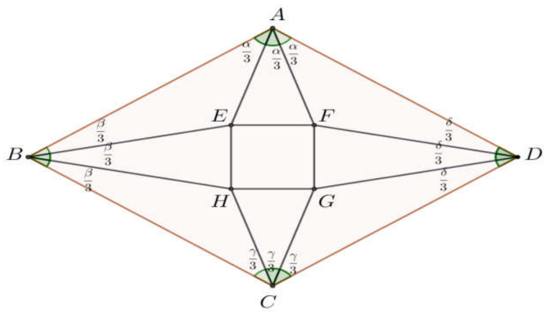

Figure 2 Morley's theorem on rhombus.

From Morley's theorem in a special quadrilateral a specialquadrilateral is formed which can be seen in Table 1.

Table 1 Morley's theorem in a special quadrilateral

No	Rectangular$ABCD$	Morley quadrilateral $EFGH$
1	Square	Square
2	Rectangle	Rhombus
3	Rhombus	Rectangle
4	Isosceles trapezoid	Kite
5	Kite	Isosceles trapezoid

RESULT

The following are the results and discussion Morley's theorem outer trisektor in any triangle and special quadrilateral.

Theorem 1

In any ΔABC, adjacent trisektor outer will intersect, if the intersection is connected then ΔDEF is formed (Figure 3) which will be proved ΔDEF is an equilateral triangle.

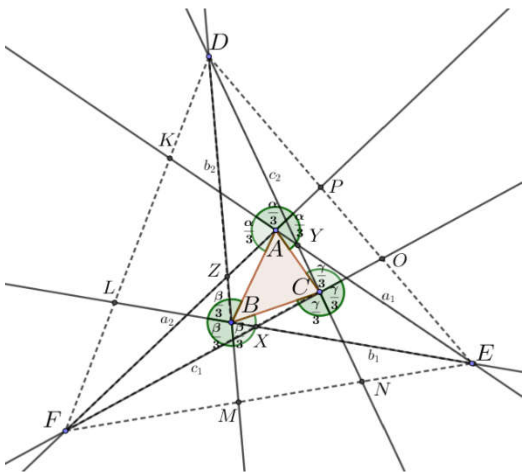

Figure 3 Development of the Morley outer trisektor theorem in any triangle ΔABC.

In any ΔABC, the adjacent outer trisektor will intersect ie between lines b_2withc_2, a_1 withb_1anda_2with c_1, Suppose a pointD, EandF are intersecting points and if all three intersection points are connected thenΔDEFis equa

Proof

It will be proven that $\Delta DEF$is an equilateral triangle showing the angle of 60^0,which will be proven using the concept of congruence. The extension of the trector will intersect, extend theBEline to the DFside, for example point$L$and extension the CFline to the DEside, for example point O, the intersection ofELandFOside, for example point$X$. Suppose the extension of theDBline to theEFside, for example point $M$and extension of the FAline to the DEside, for example point P, the intersection DMandFPside, for example point Z. Then extend the EAline to theDFfor example Kand extension DCline to the FEside, for example point N, the intersection EKandDN side, for example pointY.

Look at ΔFZMand ΔDZP, note$\angle DZP = \angle FZM$because of the alignment of $\angle DZF$.Large($\angle ZMF$ and$\angle FZM$)and($\angle ZPD$ and$\angle DZF$)are unilateral angles. Large $\angle DZP = \angle FZM$based on the unilateral angle properties then $\angle ZPD = \angle ZMF$,because two pairs of angles are equal then$\angle PDZ = \angle MFZ$.Look at ΔPDZandΔMFZ, because there are two equal angles, namely $\angle PDF = \angle MFD$dan $\angle PDZ = \angle MFZ$, then large $\angle ZDF = \angle DFZ$. To be shown using the sine rule will be shown the length of$FZ = MZ$.Based on the corner-side-corner postulate then $\Delta DPZ \cong \Delta FMZ$. In the same way $\Delta MXF \cong \Delta EXOand\Delta EYN \cong \Delta DYK$

In ΔABC, suppose that is $\angle A = \alpha$, large$\angle B = \beta$, andlarge$\angle C = 180^0 - (\alpha + \beta)$. The size of each outer trisektor at ΔABC. Outer trisektorA $= \frac{360^0 - \angle A}{3} = 120^0 - \frac{\angle \alpha}{3}$.Outer trisektorB $= \frac{360^0 - \angle B}{3} = 120^0 - \frac{\angle \beta}{3}$.Outher trisektorC $= \frac{360^0 - 18\,0^0 - (\alpha+\beta)}{3} = 60^0 - \frac{\alpha+\beta}{3}$.

Look at ΔFAEthere is $\angle FAC$large$120^0 - \frac{\angle \alpha}{3}$becauseOuter trisektor$\angle A$, $\angle BAC = \angle \alpha$, $\angle ZAB = \angle YAC$ based ongeneralisasi format-torricelli, that:

$$60^0 - \frac{2\angle \alpha}{3} = \angle ZAB = \angle YAC \qquad (1)$$

In the same way it is obtained $\angle ZBA = 60^0 - \frac{2\angle \beta}{3}$ dan $\angle XCB = -60^0 - \frac{2(\angle \alpha + \angle \beta)}{3}$. Look atΔFXE, it is clear that the large angle of the triangle amounts to 180^0, so,

$$\angle EFX = \frac{\angle \alpha}{3} \qquad (2)$$

Then in ΔDZFthe same thing is obtained

$$\angle DFZ = 60^0 - \frac{(\angle \alpha + \angle \beta)}{3} \qquad (3)$$

Look at ΔCFA, noted that$\angle FAC = \angle \alpha + \angle ZABand\angle ACF = \angle C + \angle BCX$, then:

$$\angle CFA = \frac{\angle \beta}{3} \qquad (4)$$

Large$\angle EFD$obtained by substituting equations (2), (3) and (4) into the following equation:

$$\angle EFD = \angle EFX + \angle DFZ + \angle CFA$$
$$= \frac{\angle \alpha}{3} + 60^0 - \frac{(\angle \alpha + \angle \beta)}{3} + \frac{\angle \beta}{3} = 60^0$$

In the same way obtained$\angle DEF = \angle EFD = \angle FDE = 60^0$. Because of the three corners 60^0, thenΔDEFis an equilateral triangle with any outertrisektor ΔABC.

Theorem 2

Look at $\blacksquare ABCD$isosceles trapezoid, adjacent ones intersect with each outer. If all four cut points are connected, they are formed $\blacksquare EFGH$ which can be seen in (Figure 6), which will be shown $\blacksquare EFGH$kite.

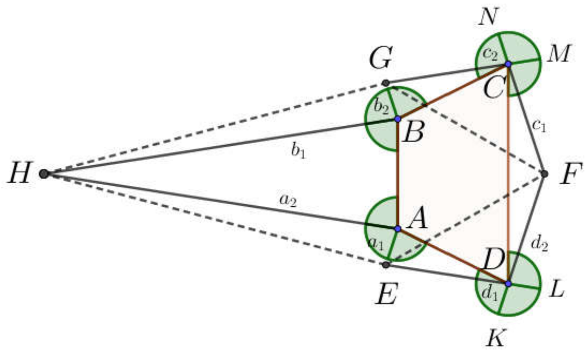

Figure 6 Morley'stheorem *outer trisektor* on isosceles trapezoid.

Look at ■$ABCD$isosceles trapezoid, *outer trisektor* adjacent ones will intersect. Suppose a point E, F, G and H is the point of intersection between the trisector lines d_1 with a_1, d_2 with c_1, b_2 with $c_2$andb_1 with a_2. If the four points are connected, a produk kite $EFGH$.

Proof

It will be proven that■$EFGH$ a kite,by showing the length of the side$EF = FG$ dan $EH = GH$and large$\angle FEH = \angle FGH$, Will be indicated by the concept congruence. Large$\angle MCF = \angle NCG$ a straightener from $\angle MCN$,then$\angle MCF = \angle NCG$,so then $\angle FCD = \angle GCB$.Known■$ABCD$ a isosceles trapezoidt$\angle BCD = \angle CDA$,so that$\angle FCG = \angle GCB = \angle FDC = \angle ADE$. So that it can be shown$\Delta ADE \cong \Delta BGC$. From these results can be shown $\Delta HGB \cong \Delta HEAand\Delta FCG \cong \Delta FDE$which results in side length $EF = FG$ and $EH = GH$. Next is shown large Known $\angle FEH = \angle FGH$. $\Delta FCG \cong \Delta FDE$, so then$\angle CGF = \angle DEF$because there are two pairs of angles that are equal, then$\angle FEH = \angle FGH$. Because of the length of the side$EF = FG$and$EH = GH$and large$\angle FEH = \angle FGH$, proven that quadrilateral EFGH akite*outer trisektor*.

CONCLUSIONS

The results of the study are the Morley's theorem outer trisektor on triangles and special quadrilateral. Morley theoremthe outer trisektor of an arbitrary triangle is an outer triangular equilateral triangleand if■ABCD kite in the form of an ■EFGHaisosceles trapezoid outer trisektor.

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