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Research Article

THE COMPARISION OF MINIMUM INDEPENT NEIGHBOURHOOD SET AND THE BONDAGE OF AN INTERVAL GRAPH G

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A R T I C L E I N F O

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A B S T R A C T

Interval graphs have a wide variant of applications to varies branches of science and technology. Among the varies applications of the theory of domination, independent neighborhood sets the most often discussed is a communication networks. This network consists of communication links between a fixed set of sides. Suppose communication network does not work due to link failure. Then the problem is, what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sides be possible. This leads to the introduction of the concepts of minimum independent neighborhood set and bondage number of a graph. In this paper we discuss the comparison of minimum independent neighborhood set and the bondage number of an interval graph G.

INTRODUCTION

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of graph theory due to its wide application to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of application to many fields like engineering, physical, social and biological sciences, linguistics ect., the theory of domination has been the nucleus of research activity in graph theory in recent time.

Domination in graphs has applications to several fields. Domination [3] arises in facility location problem, where the number of facilities like hospitals, fire stations is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that ever one is serviced. Concepts from domination [1,2] also appear in problems involving finding sets of representatives in monitoring communication or electrical networking, and in land surveying like minimizing the number of places a surveyor must stand in order to take high measure mints for an entire region.

In graph theory, an independent set or stable set is a set of vertices in a graph, no two of which are adjacent.

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That is, it is a set S of vertices such that for every two equivalently, each edge in the graph has at most one end point in S. The size of an independent set is the number of vertices it contains independent set have also been called internally stable sets.

A maximal independent set [9,10] is either an independent set such that adding any other vertex to the set forces the set to contain an edge or the set of all vertices of the graph.

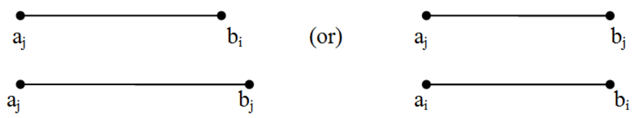
Shortest path algorithms are used in many application of everyday life. Consider using computer navigation software to obtain directions to a place you have never driven to before. In most cases, there are many paths one could talk in order to arrive at that location. This software creates a graph with the vertices representing a physical location and the edges which represent the road that connects two locations. If there is not a road between locations, then there is not an edge in the graph. Next, a weight is associated with each edge. In this example, the primary metric used for weight is distance. However, other factors in this example are considered when assigning a weight, or cost, to an edge such as traffic and average speed of vehicles on a give road.

The bondage number [4] b(G) of a nonempty graph G is the minimum cardinality among all sets of edges E for which $\sigma(G-E) > \sigma(G)$. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a "nondominating" set in the resultant spanning subgraph. Since the domination number of every spanning subgraph of a nonempty graph G is at least as great as $\sigma(G)$, the bondage number of a nonempty graph is well defined. In what follows, we investigate the value of the bondage number in progressively more general setting.

Preliminaries

Let I = {I_1,I_2,I_3,………I_n} be an interval family where each I_i is an interval on the real line and $I_i = [a_i,b_i]$ for i = 1,2,3,…..n. Here a_i is called the left end point and b_i is right end point of I_i. Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to intersect each other if they have non-empty intersection.

A graph G(V,E) is called an interval graph [7,8] if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if there corresponding intervals in I intersect. That is if i = [a_i,b_i] and j= [a_i,b_i], then i and j interest means either $a_j < b_i$ or $a_i < b_j$



Let G (V,E) be a graph. The neighborhood [5,6] of a vertex v in G is defined as the set of vertices adjacent with v (including v) and is denoted by nbd[v]. A subset S of V in G is called a neighborhood set of G if G = $\cup_{v\in s} < nbd[v] > where < nbd[v] >$ is the vertex induced subgraph of G. The neighborhood number of G is defined as the minimum cardinality of a neighborhood set of G. A neighborhood set with minimum cardinality is called a minimum neighborhood set. In addition if the set S is independent then S is called an independent neighborhood set of G.

For each interval i, let nbd[i] denote the set of intervals that intersect i (including i). Let min(i) denote the smallest interval and max(i) the largest interval in nbd[i]. The interval in nbd[i] with largest (or smallest) numeric value is called largest (or smallest) interval. Define NI(i) = j, if $b_i < a_j$ and there do not exist an interval k such that $b_i < a_k < a_j$. If there is no such j, then define NI(i) = null.

We now define Next(i) = max({nbd[min(NI(i))]}\{nbd[i]}). We may assume that there is no interval i ∈ I that intersects all other intervals in I. For {i} itself becomes a minimum neighborhood set.

First we augment I with two dummy intervals say, I_0 and I_{n+1}, where I_0 = [a_0,b_0], and I_{n+1} = [a_{n+1},b_{n+1}] such that $b_0 < \min_{1\le k\le n}\{a_k\}$ and $a_{n+1} > \max_{1\le k\le n}\{b_k\}$.

Let I_1 = I ∪ {I_0,I_{n+1}}. As in I the intervals in I_1 are also indexed by increasing order of their right endpoints, namely $b_0 < b_1 <,\ldots.< b_{n+1}$.

We now construct a directed network D (N,L) associated with G. For its nodes we take those intervals in I_1 which are not properly contained within other intervals. Because if there is an interval j which contains another interval i, then the minimum neighborhood set containing i can be changed to {MINS\i} ∪ {j}.

The lines in L are partitioned into two disjoint sets L_1 and L_2 which are defined below. For j∈ D, there is a directed line (I_0,j) between I_0 and j that belongs to L_1 if and only if there is no interval I_h such that $b_0 < a_h < b_h < a_j$. Similarly there is a directed line (j,I_{n+1}) between j and I_{n+1} that belongs to L_1 if and only if there is no interval I_h such that $b_j < a_h < b_h < a_{n+1}$. This gives the scope to join the intervals I_0 and I_{n+1} to other intervals in I and it is obvious that all such joined directed lines, belong to L_1. Next for i, j ∈ D, there is a directed line (i,j) between i and j that belongs to L_2 if and only if j = Next(i).

A sub set D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

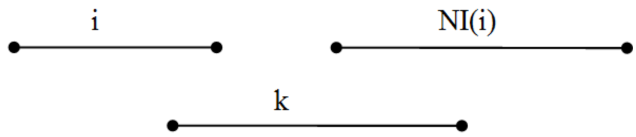
The bondage number b(G) of a non-empty graph G is the minimum cardinality among all set of edges E_1 for which $\gamma(G - E_1) > \gamma(G)$. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a non-dominating set in the resultant spanning subgraph. Since the domination number of every spanning subgraph of a non-empty graph G is at least as great as $\gamma(G)$, the bondage number of a non-empty graph is well defined.

This concept was interdoduced by Fink *et.al*[4] and they have studied this parameter for some standard, trees and general bounds are obtained.

General Conditions

First we will discuss the following general conditions to find the minimum neighborhood set or minimum shortest path of an interval graph corresponding to an interval family I={I_1,I_2,I_3,………I_n}

1. If i and k are any two intervals which are intersecting and j such that i < j < k then j intersects k.
2. If the directed line (0,j) ∈ L_1 where j is any interval of I, then the intervals between 0 and j belong to nbd[j].
3. If the directed line (j,n+1) ∈ L_1, where j is any interval of I, then the intervals between j and n+1 belong to nbd[j].
4. If i is any intervals and k = min(NI(i)) then the intervals between i and k intersect i.



5. If i is any interval I then i < min(NI(i))
6. If i, j are any two intervals in I such that j = Next(i), then i and j are non-adjacent.

Theorem.1*:* Let I = {I_1,I_2,I_3,………I_n} be an interval family and G be an interval graph corresponding to I. and if the directed line (i, j) ∈ L_2, then the intervals between i and j belong to nbd[i] or nbd[j].

Proof: Suppose G be an interval graph corresponding to an I and the directed line (i, j) ∈ L_2. Now our aim to show that the intervals between i and j belong to nbd[i] or nbd[j]. Let (i, j) ∈ L_2. Then j = next(i). Let p be any interval between i and j then it will arise four cases.

Case 1: Suppose p intersects i, in such case p ∈ nbd[i].

Case 2: Suppose p intersects min (NI(i)) and does not intersect i. Then p ∈ nbd[NI(i)]. Since p does not intersect i, p∉ nbd[i]. So nbd[min(NI(i))]\nbd[i] contains p. Since j is the maximum element in nbd[min((NI(i))] and p ∈ nbd[min(NI(i))] it follows that p must intersect j. That is p ∈ nbd[j].

Case 3: Assume that p does not intersect neither i nor min(NI(i)). Suppose i < p < min(NI(i)).Then i and min(NI(i)) intersect implies p and min(NI(i)) intersect. Suppose min(NI(i)) < p < Next(i). Again min(NI(i)) intersects Next(i) implies p and Next(i) intersect. Therefore p does not intersect neither i nor min(NI(i)) does not arise.

Case 4: Suppose p intersects j. Then clearly p ∈ nbd[j]. Thus for all possibilities, then intervals between i and j belong to nbd[i] or nbd[j].

Theorem 2: Let G be an interval graph corresponding to an interval family I = {I_1,I_2,I_3,………I_n}. Let (i,j) be any directed line in D. Then the vertex induced sub graph G' on the vertex set {I_i,I_{i+1},…I_{j-1},I_j} is a subgraph of the induced graph <nbd[I_i] ∪ nbd[I_j]>.

Proof: Let G be an interval graph corresponding to I, and G' be the induced subgraph on the vertex set {I_i,……I_j}. We know that the general conditions 2, 3 and theorem1, it's clear that the vertex set {I_i,……I_j}∈ nbd[I_i] ∪ nbd[I_j]. It suffices to show that the edge of the graph G' occur in < nbd[I_i] ∪ nbd[I_j] >. Let I_r and I_s be any two arbitrary intervals between I_i and I_j. Without loss of generality assume that I_r and I_s. Now (i, j) ∈ D implies that (i, j) ∈ L_1 or (i, j) ∈ L_2. Suppose (i, j) ∈ L_1. Then either i = 0 or j = n+1. Suppose i = 0. Then by the general condition 2 in intervals between I_0 and I_j belong to nbd[I_j]. In particular I_r , I_s ∈ nbd[I_j]. Therefore the edge (I_r , I_s) ∈ < nbd[I_j] >. Similarly when j = n+1 it follows that I_r , I_s ∈ nbd[I_i] and so the edge (I_r , I_s) ∈ < nbd[I_i] >.

Suppose that (i, j) ∈ L_2. Then by theorem 1, the intervals between I_i and I_j belong to nbd[I_i] ∪ nbd[I_j]. That is I_r, I_s ∈ nbd[I_i] ∪ nbd[I_j]. If possible, let both I_r, I_s ∈ nbd[I_i]. Then the edge (I_r, I_s) ∈ < nbd[I_i] >. Similarly if I_r, I_s ∈ nbd[I_j] then (I_r, I_s) ∈ < nbd[I_j] >.

Hence assume that I_r ∈ nbd[I_i] and I_s∈ nbd[I_j]. Again it is clear that the edge (I_r, I_s) ∈ < nbd[I_i] ∪ nbd[I_j] >. Thus for all possibilities, the edge (I_r, I_s) ∈ < nbd[I_i] ∪ nbd[I_j] > since I_r, I_s are arbitrary, if follows that G' ⊆ < nbd[I_i] ∪ nbd[I_j] >.

Therefore the theorem is proved.

Theorem 3: Let G be an interval graph and P is a shortest directed path between the vertex 0 to n+1 in D(N,L) then vertices in P other than 0 to n+1 correspond to a minimum independent neighborhood set of an interval graph.

Proof: Let P be a shortest directed path from vertex 0 to n+1 in D. Define S = {I_i: vertex i appears in P, $i \neq 0,\ i \neq n+1$}. For each directed line (i,j) in P, by general condition 2,3 and theorm1, if follows that all intermediate intervals I_{i+1},I_{i+2},….I_{j_1} between I_i and I_j belong to nbd[I_i] ∪ nbd[I_j]. Hence all intermediate intervals between the intervals in S belong to $\bigcup_{I_i,I_j \in s} < nbd[I_i] \cup nbd[I_j] >$. Since the intervals in S correspond to the vertices in path P, the intervals in between I_0 and the first interval in S as well as the intervals in between the last interval in S and I_{n+1} also belong to $\bigcup_{I_i \in s} < nbd[I_i] >$.
Thus all the vertices in G are exhausted by the vertices in S. That is V(G) = $\bigcup_{I_i \in s} nbd[I_i]$. But by theorem 2. < {$I_i$,……$I_j$} >⊆< nbd[$I_i$] ∪ nbd[$I_j$] > where I_i,I_j ∈ S.
Therefore $\bigcup_{I_i,I_j \in s} < \{I_i \dots I_j\} \subseteq \bigcup_{I_i,I_j \in s} < nbd[I_i] \cup nbd[I_j]\ >$.

Since V(G) = $\bigcup_{I_i \in s} nbd[I_i]$, it follows that G = $\bigcup_{I_i \in s} < nbd[I_i] >$.

Thus S is a neighborhood set of G. By general condition 6, the vertices in S are non-adjacent. There for S forms an independent neighborhood set of G. Since P is shortest, it follows that S is a minimum independent neighborhood set of G.

Practical Problem

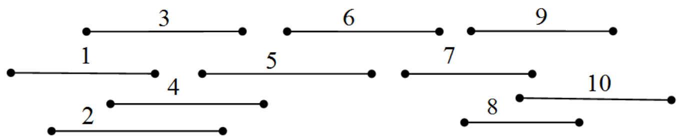


Fig 1 Interval family I

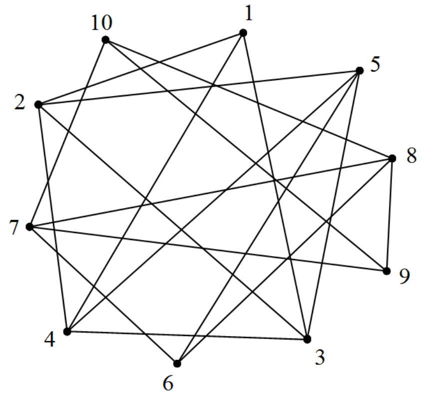


Fig 2 Interval graph G

MINIMUM INDEPENDENT NEIGHBORHOOD SET:

Nbd[1] = {1,2,3,4}	min(1) = 1	NI(1) = 5
Nbd[2] = {1,2,3,4,5}	min(2) = 1	NI(2) = 6
Nbd[3] = {1,2,3,4,5}	min(3) = 1	NI(3) = 6
Nbd[4] = {1,2,3,4,5}	min(4) = 1	NI(4) = 6
Nbd[5] = {2,3,4,5,6}	min(5) = 2	NI(5) = 7
Nbd[6] = {5,6,7,8}	min(6) = 5	NI(6) = 9
Nbd[7] = {6,7,8,9,10}	min(7) = 6	NI(7) = null
Nbd[8] = {7,8,9,10}	min(8) = 7	NI(8) = null
Nbd[9] = {7,8,9,10}	min(9) = 7	NI(9) = null
Nbd[10] = {7,8,9,10}	min(10) = 7	NI(10) = null

Next (i) = max ({nbd[min((NI(i))}\{nbd[i]})
Next (1) = max ({nbd[min((NI(1))}\{nbd[1]}) = 5
Next (2) = max ({nbd[min((NI(2))}\{nbd[2]}) = 6
Next (3) = max ({nbd[min((NI(3))}\{nbd[3]}) = 6
Next (4) = max ({nbd[min((NI(4))}\{nbd[4]}) = 6
Next (5) = max ({nbd[min((NI(5))}\{nbd[5]}) = 7
Next (6) = max ({nbd[min((NI(6))}\{nbd[6]}) = 9
Next (7) = max ({nbd[min((NI(7))}\{nbd[7]}) = null
Next (8) = max ({nbd[min((NI(8))}\{nbd[8]}) = null
Next (9) = max ({nbd[min((NI(9))}\{nbd[9]}) = null
Next (10) = max ({nbd[min((NI(10))}\{nbd[10]}) = null
Now the dummy intervals I_0 and I_{n+1} are augmented to I:

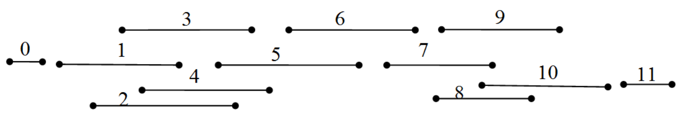


Fig 3 Interval family I_1 = I ∪ {I_0,I_{n+1}}

Directed Network D(N,L) is constructed as follows
N = {0,1,2,3,4,5,6,7,8,9,10,11}
L = $L_1 \cup L_2$

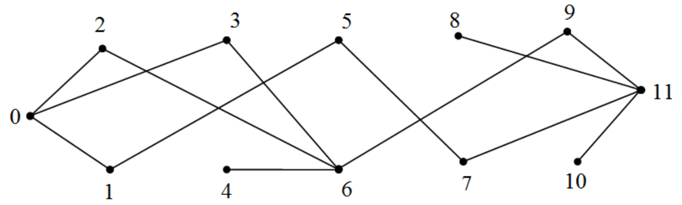


Fig 4 Networking

In the D(N,L) of the above example, observe that the shortest path from node 0 to node 11 is (0,1,5,7,11) or (0,2,6,9,11) or (0,3,6,9,11). Deleting the dummy nodes 0 and 11 from the above shortest paths we get three minimum independent neighborhood set namely (1,5,7), (2,6,9), (3,6,9) of the interval graph .

To Find A Bondage Number

***Theorem*:** Let G be an interval graph corresponding to an interval family I = {I_1,I_2,I_3,………I_n}. Let D = {x,y). Suppose x dominates S_1= {1,……..i} and y dominates S_2 = {i+1,…..,n}. Suppose there are two vertices say $z_1,z_2 \in S_1$ or S_2 such that z_1, z_2 also dominates S_1 or S_2 respectively, then the bondage number b(G) = 3.

Proof: Let I = {I_1,I_2,I_3,………I_n} be an interval family and G is an interval graph of G. Now we have to show that the bondage number b(G) = 3 from an interval graph G corresponding to an interval family I. Let D = {x,y} and x,y satisfy the hypotheses of the theorem. Suppose z_1, $z_2 \in S_1$ and z_1, z_2 also dominates S_1. Let k be an arbitrary vertex in S_1, k ≠ i,x, z_1, z_2. Now deleted the edges xk, z_1k, z_2k that are incident with k from G. If d(k) = 3, then k becomes an isolated vertex in G_1 = G− {xk,z_1k, z_2k}.

Thus D_1 = D U {K} becomes a dominating set of G_1 and since D is minimum it follows that D_1 is minimum in G_1. Hence $\gamma(G_1) > \gamma(G)$ and hence the bondage number b(G) =3.

Suppose the degree of vertex d(k)>3. Then there is at least one vertex, say j in S_1 such that j is adjacent to k and j ≠x, z_1, z_2. Let G_1 = G− {xk, z_1k, z_2k}. In G_1, k is not dominated by x, z_1, z_2, but is dominated by j, for there every vertex in S_1 other than k is dominated by x or z_1 or z_2 in G_1. Therefore every vertex in S_1 is dominated by {x,j} or {z_1,j} or {z_2,j} in G_1. Thus D_1 = D U {j} becomes a dominating set if G_1 and since D is minimum in G it follows that D_1 is also minimum in G_1. Hence the bondage $\gamma(G_1) > \gamma(G)$ so that b(G) = 3. Similarly we can show that if the vertices z_1, $z_2 \in S_2$.

In this connection our aim to show that the comparison of bondage number and a minimum independent neighborhood set of in an interval graph G. In fact we have already proved in theorem 1, theorem 2, theorem 3 of a minimum independent neighborhood set and theorem 4 the bondage number of G. As follows the practical problem of an interval family corresponding to an interval graphs G.

Practical Problem

We have already found the domination number from G. Next we will find the bondage number of G.
Dominating set D = {4,8} and $\gamma(G) = 2$.
Remove the edges (1,2), (1,3), (1,4) from G.

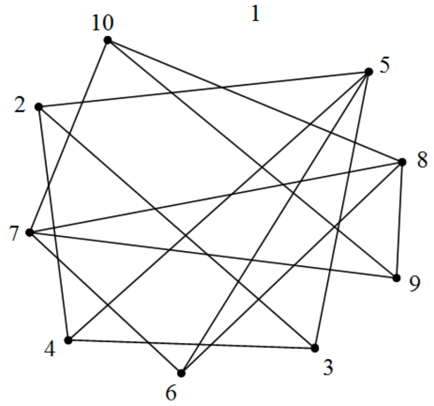


Fig 5 G_1 = G - {(1,2)(1,3)(1,4)}

Dominating set of G_1 = D_1 ={1,4,8} and $\gamma(G) = 3$
There fore $\gamma(G - e) > \gamma(G)$. Bondage number b(G) = 3.

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