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Research Article

FIXED POINT THEOREM OF WEAK COMMUTING MAPPINGS

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In this paper we prove some fixed point theorem on weak** commuting mappings of Complete metric space.

INTRODUCTION

Das and Visanathan Naik [1], have proved a theorem for two commuting mappings. Later on Fisher [3] extended and proved a common fixed point of commuting mappings already Fisher [2] proved the following theorem for commuting mappings T and S.

Theorem [F1]

If S is a mapping and T is a continuous mapping of the complete metric space X into itself and satisfying the inequality:

$D(STx, TSy) \le c\{d(Tx, TSy) + d(Sy, STx)\}$

for all x, y in X where $0 \le c \le ½$, then S and T have unique common fixed point.

The purpose of this note is prove two results concerning fixed points of weak** commuting mappings defined on complete metric space and satisfying some new functional inequality.

Definition [1]: Two self mappings S and T of metric space (X,d) is called weak** commuted, if $S(X) \subset T(X)$ and for any $x \in X$,

$d(S^2T^2x, T^2S^2x) \le d(S^2Tx, TS^2x) \le d(ST^2x, T^2Sx) \le d(STx, TSx) \le d(S^2x, T^2x)$.

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Definition [2]: A map S: $X \to X$, X being metric space, is called an idempotent if $S^2 = S$.

Example [1] : Let X = [0,1] with the Euclidean metric space and define S and T by $Sx = x/(x+3)$; $Tx = 1/3$ for all $x \in X$. Then $[0, 1/4] \subset [0, 1/3]$ where $Sx = [0, 1/4]$ opr $Tx = [0, 1/3]$

$d(S^2T^2x, T^2S^2x) = x/(2x+81) - x/(36x +81) = 32x^2/(4x+81)(36x+81)$

$\le 8x^2/(4x+27)(12x+27) = d(S^2Tx, TS^2x)$
$d(S^2Tx, TS^2x) = 8x^2/(4x+27)(12x+27) \le 8x^2/(x+27)(9x+27) = x/(x+27) - x/(9x+27) = d(ST^2x, T^2Sx)$
$d(ST^2x, T^2Sx) = 8x^2/(x+27)(9x+27) \le 2x^2/(x+9)(3x+9) = x/(x+9) - x/(3x+9) = d(STx, TSx)$
$d(STx, TSx) = 2x^2/(x+9)(3x+9) \le 4x^2/9(4x+9) = x/9 - x/(4x+9) = d(S^2x, T^2x)$

Using [0, 1] for $x \in X$ conclude that

$d(S^2T^2x, T^2S^2x) \le d(S^2Tx, TS^2x) \le d(ST^2x, T^2Sx) \le d(STx, TSx) \le d(S^2x, T^2x)$.

We have prove the following theorem

Theorem [1]: If S is a mapping and T is a continuous mapping of the complete metric space X into itself and satisfying the inequality

[1.1] {S, T} is weak** commuting pair,
[1.2] $d(S^2T^2x, T^2S^2y) \le c \max\{d(T^2x, S^2y), d(T^2x, S^2T^2x), ½(d(T^2x, T^2S^2y) + d(S^2y, S^2T^2x), D(S^2y, T^2S^2y)\}$

for all x, y in X, where $0 < c \leq 1$, then S and T have unique common fixed point.

Proof : Let x be an arbitrary point in X. Define

$(S^2T^2)^n x = x_{2n}$ or $T^2(S^2T^2)^n x = x_{2n+1}$, where n = 0,1,2, …

By contrastive condition [1.2],

$d(x_{2n}, x_{2n+1}) = d((S^2T^2)^n x, T^2(S^2T^2)^n x)$
$= d(S^2T^2(S^2T^2)^{n-1}x, T^2S^2(T^2(S^2T^2)^{n-1}x))$
$\leq c \max\{d(T^2(S^2T^2)^{n-1}x, S^2(T^2(S^2T^2)^{n-1}x), d(T^2(S^2T^2)^{n-1}x, S^2 T^2(S^2T^2)^{n-1}x), \frac{1}{2}\{d(T^2(S^2T^2)^{n-1}x, T^2 S^2(T^2(S^2T^2)^{n-1}x) + d(S^2(T^2(S^2T^2)^{n-1}x, S^2 T^2(S^2T^2)^{n-1}x)\}, d(S^2(T^2(S^2T^2)^{n-1}x, T^2 S^2(T^2(S^2T^2)^{n-1}x)\} = c \max\{d(x_{2n-1}, x_{2n}), d(x_{2n-1}, x_{2n}), \frac{1}{2}\{d(x_{2n-1}, x_{2n+1}) + d(x_{2n}, x_{2n})\}, d(x_{2n}, x_{2n+1})\}$

If $d(x_{2n}, x_{2n+1}) > d(x_{2n-1}, x_{2n})$ in the above inequality then we get

$D(x_{2n}, x_{2n+1}) \leq c\, d(x_{2n}, x_{2n+1})$, a contradiction.

Hence $d(x_{2n}, x_{2n+1}) \leq d(x_{2n-1}, x_{2n})$,

then we get $d(x_{2n}, x_{2n+1}) \leq c\, d(x_{2n-1}, x_{2n})$

Proceeding in the similar manner, $d(x_{2n}, x_{2n+1}) \leq r^{2n-1} d(x_0, x_1)$

so, $d(x_n, x_m) \leq \sum_{k=n}^{m} d(x_k, x_{k-1})$ for $m > n$

Since $c < 1$, it follows that the sequence $\{x_n\}$ is a Cauchy sequence in the complete metric space X and so it has a limit in X, that is $\lim_{n\to\infty} x_{2n} = u = \lim_{n\to\infty} x_{2n+1}$

and since T is continuous, we have $u = \lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} T^2x_{2n} = T^2u$

Further, $d(S^2u, x_{2n+3}) = d(S^2T^2u, T^2(S^2T^2)^{n+1}x,)$
$= d(S^2T^2u, T^2S^2(T^2(S^2T^2)^n x))$
$\leq c \max\{d(T^2u, S^2(T^2(S^2T^2)^n x), d(T^2u, S^2 T^2u),$
$\frac{1}{2}\{d(T^2u, T^2 S^2(T^2(S^2T^2)^n x) + d(S^2(T^2(S^2T^2)^n x, S^2 T^2u)\},$
$d(S^2(T^2(S^2T^2)^n x, T^2 S^2(T^2(S^2T^2)^n x)\}$

$= c \max\{d(T^2u, x_{2n+2}), d(T^2u, S^2 T^2u), \frac{1}{2}\{d(T^2u, x_{2n+3}) + d(x_{2n+2}, S^2 T^2u)\},$
$d(x_{2n+2}, x_{2n+3})\}$
$= c \max\{d(u, x_{2n+2}), d(u, S^2u), \frac{1}{2}\{d(u, x_{2n+3}) + d(x_{2n+2}, S^2u)\},$
$d(x_{2n+2}, x_{2n+3})\}$

making $n \to \infty$, it follows that,

$d(S^2u, u) \leq c\, d(u, S^2u)$

since $c < 1$, which implies that $d(S^2u, u) = 0$

and so $S^2u = u = T^2u$

Now weak** commutativity of pair {S, T} implies that

$S^2T^2u = T^2S^2u$; $S^2Tu = TS^2u$; $ST^2u = T^2Su$

and so $S^2Tu = Tu$ and $T^2Su = Su$.

Now $d(u, Su) = d(S^2T^2u, T^2S^2(Su))$
$\leq c \max\{d(T^2u, S^2(Su)), d(T^2u, S^2 T^2u), \frac{1}{2}\{d(T^2u, T^2 S^2(Su)) + d(S^2(Su), S^2 T^2u)\}, d(S^2(Su), T^2 S^2(Su))$
$= c\, d(Su, u)$

is a contradiction, as $c < 1$ and so $u = Su$.

Similarly we can show that u = Tu.

Hence u is common fixed point of S and T.

Now suppose that v is a second common fixed point of S and T. Then

$d(u, v) = d(S^2T^2u, T^2S^2v) \leq c \max\{d(T^2u, S^2v), d(T^2u, S^2 T^2u)$
$\frac{1}{2}\{d(T^2u, T^2 S^2v) + a_4 d(S^2v, S^2 T^2u)\},$
$d(S^2v, T^2 S^2v)\}$
$d(u, v) = c\, d(u, v)$

and since $c < 1$, it follows that $u = v$.

Hence S and T have unique common fixed point.

This complete the proof of the theorem.

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