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Research Article

INTEGRAL SOLUTIONS OF CUBIC DIOPHANTINE EQUATION $x^3 + y^3 = 16(k^2 + 3s^2)zw^2$ WITH FOUR UNKNOWNS

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A B S T R A C T

The cubic Diophantine equation given by $x^3 + y^3 = 16(k^2 + 3s^2)zw^2$ is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.

INTRODUCTION

Cubic Diophantine equations with four unknowns are rich in variety [1-3]. For an extensive review of sizable literature and various problems, one may refer [1-16]. In this communication, we consider yet another interesting cubic equation $x^3 + y^3 = 16(k^2 + 3s^2)zw^2$ and obtain infinitely many non-trivial integral solutions. A few interesting relations between the solutions and special Polygonal numbers are presented.

Notations Used

- $t_{m,n}$- Polygonal number of rank 'n' with size 'm'
- CP_n^6 - Centered hexagonal Pyramidal number of rank 'n'
- Gno_n – Gnomic number of rank 'n'
- FN_A^4– Figurative number of rank 'n' with size 'm'
- Pr_n - Pronic number of rank 'n'
- P_n^m- Pyramidal number of rank 'n' with size 'm'
- SO_n - Stella octagonal number of rank n

Method of Analysis

The Cubic Diophantine equation with four unknowns to be solved for its non zero distinct integral solutions is

$$x^3 + y^3 = 16(k^2 + 3s^2)zw^2 \quad (1)$$

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On substituting the linear transformations

$$x = u + v; \ \ y = u - v \ ; z = 2u \quad (2)$$

in (1), it leads to

$$u^2 + 3v^2 = 16(k^2 + 3s^2)w^2 \quad (3)$$

We obtain different patterns of integral solutions to (1) through solving (3) which are illustrated as follows:

Pattern I

Assume

$$w = w(a, b) = a^2 + 3b^2 = (a + i\sqrt{3}\, b)(a - i\sqrt{3}\, b) \quad (4)$$

where a and b are non-zero distinct integers

Write 16 as

$$16 = (2 + i2\sqrt{3})(2 - i2\sqrt{3}) \quad (5)$$

$$u^2 + 3v^2 = (u + i\sqrt{3}\, v)(u - i\sqrt{3}\, v) \quad (6)$$

Using (4), (5) & (6) in (3) and applying the method of factorization, define Equating the real and imaginary parts, we have

$$u = k(2a^2 - 6b^2 - 12ab) + s(-6a^2 + 18b^2 - 12ab) \quad (7)$$

$$v = k(2a^2 - 6b^2 + 4ab) + s(2a^2 - 6b^2 - 12ab) \quad (8)$$

Hence in view of (2), the values of x, y, z are given by

$$\left.\begin{aligned} x = x(k,s,a,b) &= k(4a^2 - 12b^2 - 8ab) + s(-4a^2 + 12b^2 - 24ab) \\ y = y(k,s,a,b) &= k(-16ab) + s(-8a^2 + 24b^2) \\ z = z(k,s,a,b) &= k(4a^2 - 12b^2 - 24ab) + s(-12a^2 + 36b^2 - 24ab) \\ w = w(k,s,a,b) &= a^2 + 3b^2 \end{aligned}\right\} \quad (9)$$

Thus (9) represent integral solutions of (1)

Properties

1. $x(k,s,a,b)+y(k,s,a,b)-z(k,s,a,b)\equiv 0$
2. $y(-1,1,a,a(a+1))-x(-1,1,a,a(a+1))-64P_a^5\equiv 0$
3. $x(1,1,a,1)+z(3,1,a,1)\equiv 0\,(mod\ 16)$
4. $x(1,1,a,1)-2y(1,1,a,1)-t_{4,4a}+24\equiv 0$
5. $y(k,s,a,a)+k\,t_{4,4a}-s\,t_{4,4a}\equiv 0$

Pattern II

Rewrite (3) *as*

$u^2+3v^2=16(k^2+3s^2)w^2*1$

Write 1 as

$$1=\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \quad (10)$$

Following the procedure similar to pattern-I, the corresponding non-zero distinct integral solutions of (1) are found to be

$$u=\frac{1}{2}(k(-4a^2+12b^2-24ab)+s(-12a^2+36b^2+24ab))$$
$$v=\frac{1}{2}(k(4a^2-12b^2-8ab)+s(-4a^2+12b^2-24ab))$$

The corresponding non zero distinct solutions of (1) are given by

$$\left.\begin{array}{c} x=x(k,s,a,b)=k(-16ab)+s(-8a^2+24b^2) \\ y=y(k,s,a,b)=k(-4a^2+12b^2-8ab)+s(-4a^2+12b^2+24ab) \\ z=z(k,s,a,b)=k(-4a^2+12b^2-24ab)+s(-12a^2+36b^2+24ab) \\ w=w(k,s,a,b)=a^2+3b^2 \end{array}\right\} \quad (11)$$

Properties

1. $x(k,s,a,2a^2-1)+16kSO_a+8s\,t_{4,b}-24s\,t_{4,2a^2-1}\equiv 0$
2. $z(1,1,1,b)-48\,t_{4,b}+16\equiv 0$
3. $x(1,1,1,b)+w(k,s,1,b)-48t_{4,b}\equiv 0(mod\ 16)$
4. $x(1,1,a,1)+w(k,s,a,1)\equiv 48(mod\ 16)$
5. $[x(k,s,a,b)+y(k,s,a,b)]^2-z^2(k,s,a,b)\equiv 0$

Pattern III

Instead of (5) , write 16 as

$$16=\frac{(4+i4\sqrt{3})(4-i4\sqrt{3})}{4}$$

Following the procedure similar to pattern-I, the corresponding non-zero distinct integral solutions of (1) are found to be

$$u=\frac{1}{2}(k(4a^2-12b^2-24ab)+s(-12a^2+36b^2-24ab))$$
$$v=\frac{1}{2}(k(4a^2-12b^2+8ab)+s(4a^2-12b^2-24ab))$$

The corresponding non zero distinct solutions of (1) are given by

$$\left.\begin{array}{c} x(k,s,a,b)=x=k(4a^2-12b^2-8ab)+s(-4a^2+12b^2-24ab) \\ y=y(k,s,a,b)=k(-16ab)+s(-8a^2+24b^2) \\ z(k,s,a,b)=z=k(4a^2-12b^2-24ab)+s(-12a^2+36b^2-24ab) \\ w(k,s,a,b)=w=a^2+3b^2 \end{array}\right\} \quad (12)$$

Properties

1. $x(5,3,2,b(b+1))+24(Pr_b)^2+t_{4,15b}-t_{4,b}-224b=22$
2. $z(1,1,a,a(a+1))+8t_{4,a}+96\,P_a^5-24(Pr_a)^2\equiv 0$
3. $y(-1,1,a,1)+8\,t_{4,a}\equiv 24(mod\ 16)$
4. $x(-1,1,3,b)-z(-1,1,3,b)\equiv 0\,(mod\ 96)$
5. $x(6,0,1,1)-z(6,0,1,1)$ is a nasty number

CONCLUSION

To conclude, one may search for other patterns of solutions and their corresponding properties.

References

Journal Articles

1. GopalanMA, SangeetheG. *On the Ternary Cubic Diophantine Equation* $y^2=Dx^2+z^3$ Archimedes *J.Math*, 2011,1(1):7-14.
2. Gopalan MA, Vijayashankar A, Vidhyalakshmi S. *Integral solutions of Ternary cubic Equation,* $x^2+y^2-xy+2(x+y+2)=(k^2+3)z^2$, *Archimedes J.Math*,2011;1(1):59-65.
3. Gopalan M.A, Vidhyalakshmi S, Kavitha A, *Observation on the Ternary Cubic Equation* $x^2+y^2+xy=12z^3$ Antarctica *J.Math*, 2013; 10(5):453-460.
4. Gopalan M.A, Geetha D*, Lattice points on the Hyperboloid of two sheets* $x^2-6xy+y^2+6x-2y+5=z^2+4$ Impact *J.Sci.Tech*, 2010, 4, 23-32.
5. GopalanM.A, VidhyalakshmiS, KavithaA, *Integral points on the Homogenous Cone* $z^2=2x^2-7y^2$, The Diophantus *J.Math*, 2012,1(2) 127-136.
6. Gopalan M.A, Vidhyalakshmi S, Sumathi G, *Lattice points on the Hyperboloid of one sheet* $4z^2=2x^2+3y^2-4$, *The Diophantus J.Math,*2012,1(2),109-115.
7. GopalanM.A, VidhyalakshmiS, LakshmiK, *Integral points on the Hyperboloid of two sheets* $3y^2=7x^2-z^2+21$, Diophantus *J.Math*, 2012,1(2),99-107.
8. GopalanM.A, VidhyalakshmiS, MallikaS, *Observation on Hyperboloid of one sheet* $x^2+2y^2-z^2=2$ *Bessel J.Math*, 2012, 2(3), 221-226.
9. GopalanM.A, VidhyalakshmiS, Usha Rani T.R, MallikaS, *Integral points on the Homogenous cone* $6z^2+3y^2-2x^2=0$ Impact *J.Sci.Tech*, 2012,6(1),7-13.
10. GopalanM.A, VidhyalakshmiS, LakshmiK, *Lattice points on the Elliptic Paraboloid,* $16y^2+9z^2=4x^2$ *Bessel J.Math*, 2013, 3(2), 137-145.
11. Gopalan M.A, Vidhyalakshmi S, Umarani J, *Integral points on the Homogenous Cone* $x^2+4y^2=37z^2$, *Cayley J.Math*, 2013,2(2),101-107.
12. MeenaK, Vidhyalakshmi S, Gopalan M.A, PriyaK, *Integral points on the cone* $3(x^2+y^2)-5xy=47z^2$, *Bulletin of Mathematics and Statistics and Research*, *2014*,2(1),65-70.
13. GopalanM.A, VidhyalakshmiS, NivethaS, *on Ternary Quadratic Equation* $4(x^2+y^2)-7xy=31z^2$ Diophantus *J.Math*, 2014,3(1),1-7.

14. GopalanM.A, Vidhyalakshmi S, Shanthi J, *Lattice points on the Homogenous Cone* $8(x^2+y^2)-15xy=56z^2$ *Sch Journal of Phy Math Stat*, 2014, 1(1), 29-32.
15. Meena K, Gopalan M.A, Vidhyalakshmi S, Manjula S, Thiruniraiselvi, N, *On the Ternary quadratic Diophantine Equation* $8(x^2+y^2)+8(x+y)+4=25z^2$, *International Journal of Applied Research*,2015,1(3),11-14.

Reference Books

1. Dickson IE, Theory of Numbers, vol 2. Diophantine analysis, New York, Dover, 2005
2. MordellLJ. Diophantine Equations Academic Press, NewYork,1969
3. Carmichael RD, The Theory of numbers and Diophantine Analysis, NewYork, Dover, 1959.

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