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Research Article

ACCELERATED DEGRADATION TEST FOR SIMPLE STEP-STRESS MODEL USING INVERSE GAUSSIAN PROCESS

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A B S T R A C T

The accelerated Degradation testing (ADT) experiments are important technical methods in reliability studies. Different type of accelerating degradation models have developed with the time and can be used in different types of situations. However, it has become necessary for the manager to test how many no of units should be tested at a particular stress level so that the cost of testing is less. Such experiments allow the experimenter to run the test units at higher-than-usual stress conditions in order to secure failures more quickly.The Inverse Gaussian process is flexible in incorporating random effects and explanatory variables. The different types of models based on IG process are random drift model, random volatility model and random drift- volatility model. In this paper we have considered random drift model for the study onstochastic degradation models for simple step-stress model using inverse Gaussian process observed in degradation problems.

INTRODUCTION

Accelerated Degradation testing (ADT) are preferred to be used in mechanized industries to obtain the required information about the reliability of product components and materials in a short period of time. Accelerated test conditions involve higher than usual pressure, temperature, voltage, vibration, etc or any other combination of them. Data collected at such accelerated conditions are extrapolated through a physically suitable statistical model to estimate the lifetime distribution at design condition stress the life data collected from the high stresses the need to be extrapolated to estimate the life distribution under the normal-use condition. A special class of the ADTis the step-stress testing which regularly increases the stress levels at some pre-fixed time points until the test unit fails.

The main purpose of performing such test is to gather reliability information quickly or to save time as well as money. The degradation process is most often hastened under several stresses therefore we can use accelerated degradation test (ADT) to quickly obtain degradation phenomenon. In a simple constant stress ADT experiment no of units are allocated to several stress level and the degradation level of these units are measured, analyzed, and extrapolated to the failure threshold so as to estimate the life characteristics of interest under use conditions. ADTs are able to greatly abridge the testing duration, and have attracted much attention. These are two classes of models for ADT data.

The first passage time of Brownian motion is distributed as inverse Gaussian, it is logical to use it as a life time model. It is useful in studying the life testing and reliability of a product, device or subcomponent. To predict the reliability of recently developed product engineers adopt accelerated tests in order to abridge the life of the product or accelerate the degradation of their performance. During this test the products are exposed to extreme conditions such as combination of random vibrations, increases temperature, voltage or pressure. Inverse Gaussian process is useful as a repair time model. Moreover the field of reliability, the inverse Gaussian distribution has been used in a wide range of applications which includes many various fields such as cardiology, hydrology, demography, linguistic and finance has discussed byChikkara and Folks (1989).

REVIEW OF LITERATURE

Refer to early work on degradation models can be establish in Nelson (1990), While more recent work is referenced by Bagdonavicius and Nikuline (2002). In particular, degradation models based on Gaussion or other Stochastic process have been considered recently by Doksum and Normand (1995), Lu (1995), Whitmore (1995), Whitmore and Schenkelberk (1997),

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Whitemore, Crower and Lawless (1998), Bagdonavicius and Nikuline (2000). Also assuming degradation modelled by a Gaussian process with positive drift, Pettit and Young (1999) Developed by Bayesian Inference procedures for data which included both the lifetimes of items measured only at the ending of the test period. Including Lu and Meeker (1998), Boulanger and Escobar (1994), Hemada (1995), Meeker, Escobarand Lu (1998), and Meeker and Escobar (1998). Specific applications of degradation models have been reporting also by several investicators is See Carey and Koeing, (1991), Yanagisawa (1997), Meeker, Escobar (1998),

Inverse Gaussian Process Model

An inverse Gaussian process $\{Y(t);\ t \geq 0\}$ with mean function $\Lambda(t)$ and scale parameter λ has the following properties:

Y(t) has independent increments for every pair of disjoint intervals $(t_1, t_2), (t_3, t_4)$ with $t_1 < t_2 < t_3 < t_4$, the random variables $Y(t_2) - Y(t_1)$ and $Y(t_4) - Y(t_3)$are independent

Each increment $Y(t) - Y(s)$ has an inverse Gaussian distribution $IG(\Delta\Lambda(t), \lambda\,\Delta\Lambda(t)^2)$ where $\Delta\Lambda = \Lambda(t) - \Lambda(s)$ and the PDF of an inverse Gaussian distribution random variable $IG(\mu, \lambda)$ with mean μ and variance $\frac{\mu^3}{\lambda}$has discussed by Chikkara and Folks(1989) is

$$f(x;\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi}}\,x^{-\frac{3}{2}} exp\left(-\frac{\lambda\,(x-\mu)^2}{2\mu^2 x}\right)\ \ X > 0 \qquad \cdots (1)$$

$Y(0) = 0$With probability one. When the amount of degradation reaches a pre-specified critical level D, failure occurs. Let $T = Inf\{t: Y(t) = D\}$ denote the failure time. Since the inverse Gaussian process has a failure time distribution by

$$P(T < t) = P(Y(t) > D) = 1 - G(D;\ \Lambda(t), \lambda\Lambda(t)^2)$$

$$= \Phi\left[\sqrt{\frac{\lambda}{D}}(\Lambda(t) - D)\right] - e^{2\lambda\Lambda(t)}\Phi\left[\sqrt{\frac{\lambda}{D}}(\Lambda(t) + D)\right][-\sqrt{\lambda}D(\Lambda(t) + D)] \qquad \cdots (2)$$

Where $G\ (.;\ \Lambda, \lambda)$is a cumulative distribution function (CDF) of $IG(\Lambda, \lambda)$ and $\Phi$is the standard normal cdf. From above equation we can write the CDF of the failure time distribution as

$$H_\lambda(t) = \Phi\left[\sqrt{\frac{\lambda}{D}}(t - D)\right] - e^{2\lambda t}\Phi\left[\sqrt{\frac{\lambda}{D}}(t + D)\right] \qquad \cdots (3)$$

It is an increasing function. Thus, within this class of models, there is a one to one relationship between $\Lambda(t)$ and the cdf of the failure time distribution $H_\lambda(t)$ for a fixed scale parameter λ.

$$f(x;\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi}}\,x^{\frac{3}{2}} exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right) \qquad \cdots (4)$$

Where $\mu > 0$ and $\lambda > 0$ the parameter μ is the mean of the distribution and λ is a scale parameter. (Tweedie) gives three form of above pdf, which he obtained by replace the set of parameters (μ, λ) by $(\propto, \lambda)$ or (μ, ϕ), or (ϕ, λ) using the relationship given by

$$\mu = \frac{\lambda}{\phi} = (2\propto)^{\frac{-1}{2}} \qquad \cdots (5)$$

Both μ and λ are of the same physical extent as the random variable X itself; but the parameter $\mu = \frac{\lambda}{\phi}$ is invariant under a scale transformation of X as can be seen from the following relationship: `

$$f(x;\ \mu,\lambda) = \mu^{-1} f\left(\frac{x}{\mu}; 1, \phi\right) = \lambda^{-1} f\left(\frac{x}{\mu}; \phi, 1\right) \qquad \cdots (6)$$

The probability density can be numerically computed using any of the three forms in above equation as shown above the cumulative distribution function depends fundamentally on only two variables, which might be taken as $x\mu$ and ϕ. According, the case $\mu = 1$ for the (μ, ϕ) parametric form of above equation could be adopted as a standard form. This has also been obtained as a limiting form of the distribution of the sample size in a Wald's sequential probability ratio test and is sometimes referred to as the standard Wald's distribution of thedensity function model is,

$$\mu\left[\left(1 + \frac{9}{4\phi}\right)^{\frac{1}{2}} - \frac{3}{2\phi}\right] \qquad \cdots (7)$$

Inverse Gaussian Process with Random Effects

Random effects are needed in Inverse Gaussian process to account for inexplicable heterogeneous degradation rates within the product population. By linking to the Weiner process this investigates different options to incorporate the random effects in the IG process model. Consider the wiener process $W(x) = \mu x + \lambda B(x)$ where $\mu > 0$ is the drift parameter and $\lambda > 0$ is the volatility parameter and $B(x)$ is the standard Brownian motion. Given a fix threshold $\Lambda > 0$, it is well known that the first passage time $T_A = \inf\{x > 0 | W(x) \geq \Lambda\}$ follows $IG\left(\frac{\Lambda}{\mu}, \frac{\Lambda^2}{\lambda^2}\right)$going one step further, we consider a series of the thresholds $\Lambda(t)$ indexed by t with $\Lambda(0) = 0$ and $\Lambda(t)$ increasing in t, and define the first passage time process $Y(t) = T_{\Lambda(t)}$ It is easily verified that the induced $\{Y(t); t > 0\}$ is an IG process with the mean function $\frac{\Lambda(t)}{\mu}$ and variance function $\frac{\Lambda(t)}{\lambda^2}$ by asset of the stationary and independent increment property of the Wiener process $W(x)$.

The inverse relation between the IG and the Wiener processes motivates investigation of the IG process from a new perspective. Existing results on the Wiener processes can let somebody use support to the development of IG process model with the random effects. The random effect model is described below

Random Volatility Model

Consider a Wiener process $W(x) = \mu^{-1}x + \lambda^{\frac{-1}{2}}B(x)$ with the induced IG process other way of introducing unit-specific random effects is to assume that each unit possess a separate realization of the volatility parameter. Accordingly volatility parameter in the Inverse Gaussian process is random. With the random volatility parameter in the Inverse Gaussian process all units have the same mean degradation path, even though they will have different variance functions. The Inverse Gaussian process with random volatility parameter was originally proposed by Wang and Xu (2010)

Shortcoming of random volatility model is unusual to use the volatility parameter to control heterogeneity in the Weiner process thus application of random volatility model is limited. Thus random drift model was proposed which overcome inadequacy of random volatility model.

Random Drift Model

An effective way to incorporate random effect in the IG process is to let μ be a random variable. To avoid the negative values of μ (Whitmore 1986) and ensure mathematical tractability, we assume $\mu - 1$ follows a truncated normal distribution $TN\ (\omega, k^{-2}), k > 0$ with PDF

$$g(\mu^{-1};\ \omega, k^{-2}) = \frac{k.\phi[k(\mu - 1 - \omega)]}{1 - \Phi(-k\omega)}\mu > 0 \qquad \cdots (8)$$

Where $(.)$ is a standard normal PDF. In a degradation test, if the degradation of the i[th] testing unit is observed at time $t_{io} < t_{i1} < \cdots . < t_{ini}$with observations $Y_i(t_{ij})$, $j = 0,1,2, \ldots, n_i$ the joint PDF of $Y_i = [Y_i(t_{i1}), Y_i(t_{i2}), \ldots . Y_i(t_{i\,ni})]$ is computed by first conditioning on the random drift parameter μi and then marginalizing it, which yields the following equation

$$f_{IG}(Y_i) = \frac{1 - \phi(-\tilde{\omega}_i \tilde{k}_i)}{1 - \phi(-k\omega)} \frac{k}{\tilde{k}_i}\ \Pi_{j=1}^{ni} \sqrt{\frac{\lambda \Lambda_{ij}{}^2}{2\pi yij^3}} \frac{\tilde{k}_i{}^2 \tilde{\omega}_i - k^2\omega^2}{2} - \lambda \sum_{j=1}^{ni} \frac{\Lambda ij^2}{2yij} \qquad \cdots (9)$$

Where $Y_{ij} = Y_i(t_{ij}) - Y_i(t_{ij} - 1)$ is the observed increment $\Lambda_{ij} = \Lambda(t_{ij}) - \Lambda(t_{ij} - 1)$

$$\tilde{k_{ij}} = \sqrt{\lambda Y_{ij}(t_{ij}k_j) + k^2} \qquad \cdots (10)$$

$$\tilde{\omega_{ij}} = \frac{\lambda \Lambda(t_{ij}k_j) + k^2 exp \propto_0 + \propto_1 x_j}{\tilde{k}_{ij}{}^2} \qquad \cdots (11)$$

$$\tilde{\omega_{ij}} = \frac{[\lambda \Lambda(t_{ij}k_j) + k^2 exp \propto_0 + \propto_1 x_j]}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} \qquad \cdots (12)$$

Then the log-likelihood function is given by

$$l(\theta) = \sum_{i=1}^{j} \sum_{j=1}^{N_j} \left[ln \frac{k}{\tilde{k}_{ij}} + \frac{\tilde{k}_{ij}{}^2 \tilde{\omega}_{ij}{}^2 - k^2 exp(2 \propto_0 + 2 \propto_1 x_j)}{2} + \frac{1}{2} \sum_{k=1}^{k_j} \left[In(\lambda \partial \Lambda_{ijk}) - \frac{\lambda \Lambda_{ijk^2}}{y_{ijk}} \right] \right] \qquad \cdots (13)$$

$$l(\theta) = \sum_{i=1}^{j}\sum_{j=1}^{N_j}\left[ln \frac{k}{\sqrt{\lambda Y_{ij}(t_{ij}k_j) + k^2}} + \frac{\frac{\left(\lambda \Lambda(t_{ij}k_j) + k^2 exp(\propto_0 + \propto_1 x_j)\right)^2 - k^2 exp(2\propto_0 + 2\propto_1 x_j)}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)}}{2} + \frac{1}{2}\sum_{k=1}^{k_j}\left(In(\lambda \partial \Lambda_{ijk}) - \frac{\lambda \Lambda_{ijk^2}}{y_{ijk}}\right)\right] \quad \cdots (14)$$

The log likelihood function up to a constant can be expressed by the above equation. Where θ is a parameter vector include$\propto_0$, $\propto_1$, $\lambda, \beta, and\ k$.

Accelerated Degradation Test Assumptions

Let total N number of units is put into test. Suppose S_0 be the usage stress $S_H$being the maximum acceptable stress. To collect the degradation data timely we allocate these units J stress level $S_1 < 47$

$S_2 < \cdots \dots \dots \dots < S_J$ with $S_0 < S_1$and $S_J = S_H$ consider N_j units to be allocated to jth stress level. $j = 1, 2, 3, \dots . . J$. The degradation of these units is effected by the stress. Here, we have assumed $\mu_i = h(s)$, and λ is constant over s, where $h(s)$ is a link function reflecting the effect of the stress on the degradation process. Due to the above assumption the degradation speed and drift changes with the stress. Another alternative is that$\lambda = h(s)$ while μ is constant which is not valid for random drift model since $\mu$is changing from unit to unit. For simplicity and without loss of generality, the additional assumptions is, The measurement time interval, and the number of measurement $K_j$under the j[th] stress level, where $j = 1, 2, \dots \dots \dots J$, are pre-determined and The link function follows one of the following acceleration relations:

Power law relation$h(s) = \varphi_0 . s^{\propto}$
Arrhenius relation $h(s) = \varphi_0 . e^{\frac{-\propto}{s}}$
Exponential relation $h(s) = \varphi_0 . e^{\propto s}$

In real time applications the time approved for the test is often given by manager and time intervals at which the units are measured are predetermined because of the working time of experimenters. Thus, we assume that τ_j and k_j are given. In our model we delight these two variables as decision variables, and then we optimally determine their values. When the assumed stress-degradation relation i.e., is correct we can use a two-stress ADT, i.e.,$J = 2$ in our model. But, in this minimum variance plan we are unable to check the validity of the assumed stress-degradation relationship.

Thus we prefer to use three-stress ADT planning taking $J = 3$ to check the validity of the assumed model. In our settings, the purpose of ADT planning is to optimally determine the stress levels (S_j), and the number of samples for each stress level (N_j) are be investigated in our proposed work.

Normalizing the Stress Level

We standardize the stress levels depending on the acceleration relationship of the stress on the rate of degradation as follows:

$Z_j = \frac{ln\, S_j - \ln\, S_0}{ln\, S_H - \ln\, S_0}$For the power law relation

$Z_j = \frac{\frac{1}{S_0} - \frac{1}{S_j}}{\frac{1}{S_0} - \frac{1}{S_H}}$For the Arrhenius relation

$Z_j = \frac{S_j - S_0}{S_H - S_0}$For the exponential relation

From the above consistency, it is readily seen that $x_0 = 0, x_j = 1$, and $0 < Z_j \leq 1$ for $j = 1, 2 \dots . , J$. then

$$h(x) = exp\ (\propto_0 + \propto_1 Z_j)$$

$$h(x) = exp\left[ln\ \varphi_0 - \frac{\propto}{S_0} + \propto\left(\frac{1}{S_0} - \frac{1}{s}\right)\right]$$

$$h(x) = exp\left[ln\ \varphi_0 - \frac{\propto}{s}\right]$$

$$h(x) = \varphi_0 . e^{\frac{-\propto}{s}}$$

$$\ln\ h(x) = \ln \varphi_0 - \frac{\propto}{s}$$

$$\omega_j = exp\ (\propto_0 + \propto_1 Z_j) \quad \cdots (15)$$

Where

$\propto_0 = \ln\ \varphi_0 - \frac{\propto}{S_0}$, $\propto_1 = \propto (\frac{1}{S_0} - \frac{1}{S_H})$ For the Arrhenius function
$\propto_0 = \ln\ \varphi_0 + \propto ln\, S_0$, $\propto_1 = \propto (\ln\ S_H - ln\, S_0)$ For the power law function
$\propto_0 = \ln\ \varphi_0 + \propto S_0$, $\propto_1 = \propto (S_H - S_0)$ For the exponential function

Inferential Procedure

We suppose that the ith unit under the jth stress level is measured at time $t_{ijk} = k_{\tau j}$ with observations $Y_{ij}\ (t_{ijk}), k = 0, 1, \dots \dots . . , k_j$. Let $Y_{ijk} = Y_{ij}\ (t_{ijk}) - Y_{ij}\ (t_{ij}, k - 1)$ be the observed increments, and $\Lambda_{ijk} = \Lambda(t_{ijk}) - \Lambda(t_{ijk}, k - 1)$. Now, the log-likelihood function up to a constant can be expressed by the equation above 1. The Fisher information matrix $I(\theta)$ for the element $\propto_0, \propto_1, k, \omega, \Lambda(.)$ can be developed as below. We assume nonlinear function for $\Lambda(.)$, i.e., $\Lambda(t) = t_{\beta}$ and then

$\theta = (k, \delta, \propto_0, \propto_1, \beta)$' detailed expression for the elements along with the elements of the fisher information matrix can be developed as follows

$$\frac{\partial l(\theta)}{\partial \delta_j} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[0 + \frac{1}{2}\left\{\frac{2\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)k^2}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)} - 2k\omega_j{}^2\right\} + \frac{1}{2}\sum_{k=1}^{k_j}(0-0)\right]$$

$$\frac{\partial l(\theta)}{\partial \delta_j} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\frac{1}{2}\left\{\frac{2\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)k^2}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)} - 2k\delta_j{}^2\right\}\right]$$

$$\frac{\partial l(\theta)}{\partial \delta_j}\sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left\{\frac{k^2\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)}2k^2\delta_j\right\}\right] \qquad \cdots (16)$$

$$\frac{\partial^2 l(\theta)}{\partial \delta_{j^2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left\{\frac{-k^2(0 + k^2)}{\left(\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)\right.} - k^2\right\}\right]$$

$$\frac{\partial^2 l(\theta)}{\partial \delta_{j^2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left(\frac{k^4 - k^4 - k^2\left(\lambda Y_{ij}\left(t_{ij}k_j\right)\right)}{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2} - k^2\right)\right]$$

$$\frac{\partial^2 l(\theta)}{\partial \delta_{j^2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left(\frac{-k^2\left(\lambda Y_{ij}\left(t_{ij}k_j\right)\right)}{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2} - k^2\right) \qquad \cdots (17)$$

$$\frac{\partial l(\theta)}{\partial k} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left(\frac{1}{\frac{k}{\sqrt{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2}}}\frac{\sqrt{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2 - k}\frac{2k}{2\sqrt{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2}}}{\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2}\right)\right.$$

$$\left. + \frac{1}{2}\left\{\left(\frac{2\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)k\delta - \left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)^2(0 + 2k)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)^2}\right) - 2k\delta_j{}^2\right\} + 0\right]$$

$$\frac{\partial l(\theta)}{\partial k} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\frac{\lambda Y_{ij}\left(t_{ij}k_j\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)} + \left\{\frac{\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)\left\{\left(2k\delta\lambda Y_{ij}\left(t_{ij}k_j\right)\right) + 2k^3\delta_j - k\lambda\Lambda\left(t_{ij}k_j\right) - k^3\delta_j\right\}}{\left(Y_{ij}\left(t_{ij}k_j\right) + k^2\right)^2} - k\delta_j{}^2\right\}\right] \qquad \cdots (18)$$

$$\frac{\partial^2 l(\theta)}{\partial k^2} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left(\frac{-2\lambda Y_{ij}\left(t_{ij}k_j\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right) + k^2\right)^2}\right) + \frac{\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\right)^2\left\{\left(2\delta_j\lambda^2\Lambda\left(t_{ij}k_j\right)Y_{ij}\left(t_{ij}k_j\right) + 6k^2\delta_{j^2}\lambda Y_{ij}\left(t_{ij}k_j\right)\right) + 6k^2\delta_{j^2}\lambda\Lambda\left(t_{ij}k_j\right) - 3k^2\delta_j\right\}}{\left(Y_{ij}\left(t_{ij}k_j\right) + k^2\right)^4}\right.$$

$$\left. + \frac{3k^3 + 4k\lambda Y_{ij}\left(\lambda\Lambda\left(t_{ij}k_j\right) + k^2\delta_j\right)\left(2k\delta_j\lambda\right)Y_{ij}\left(t_{ij}k_j\right) + 2k^3\delta_j - k\lambda\Lambda\left(t_{ij}k_j\right) - k^3\delta_j}{\left(Y_{ij}\left(t_{ij}k_j\right) + k^2\right)^4} - \delta_{j^2}\right] \qquad \cdots (19)$$

$$\frac{\partial l(\theta)}{\partial \lambda}\sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\frac{1}{\frac{k}{\sqrt{\lambda Y_{ij}(t_{ij}k_j)+k^2}}}-\frac{k}{2}\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^{-\frac{3}{2}}.Y_{ij}(t_{ij}k_j)\right.$$

$$+\frac{1}{2}\left\{\frac{\left(\lambda\Lambda(t_{ij}k_j)+k^2\right)^2\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta_{j^2}\right)\Lambda(t_{ij}k_j)-\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta_j\right)^2 Y_{ij}(t_{ij}k_j)}{\left(Y_{ij}(t_{ij}k_j)+k^2\right)^4}\right\}$$

$$\left.+\frac{1}{2}\sum_{k=1}^{k_j}\left(\frac{1}{\lambda\Lambda_{ijk^2}}\Lambda_{ijk^2}-\frac{\Lambda_{ijk^2}}{Y_{ijk}}\right)\right]\frac{\partial l(\theta)}{\partial \lambda}$$

$$=\sum_{J=1}^{j}\sum_{i=1}^{N_j}\left[\frac{-1}{2}\frac{\lambda Y_{ij}(t_{ij}k_j)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)}\right.$$

$$+\frac{1}{2}\left\{\frac{\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta_{j^2}\right)\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)2\Lambda(t_{ij}k_j)-Y_{ij}(t_{ij}k_j)\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta_j\right)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}\right\}2$$

$$\left.+\frac{1}{2}\sum_{k=1}^{k_j}\left(\frac{1}{\lambda}-\frac{\Lambda_{ijk^2}}{Y_{ijk}}\right)\right] \quad \cdots (20)$$

$$\frac{\partial^2 l(\theta)}{\partial \lambda^2}=\sum_{J=1}^{j}\sum_{i=1}^{N_j}\left[\frac{1}{2}\frac{Y_{ij}(t_{ij}k_j).Y_{ij}(t_{ij}k_j)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}+\frac{1}{2}\left\{\frac{2\Lambda(t_{ij}k_j)+\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)-\Lambda(t_{ij}k_j)-\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta\right)Y_{ij}(t_{ij}k_j)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}\right\}\right.$$

$$-\left\{\frac{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2 Y_{ij}(t_{ij}k_j)\Lambda(t_{ij}k_j)-\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta\right)2\lambda Y_{ij}(t_{ij}k_j)+2\lambda Y_{ij}(t_{ij}k_j)k^2}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}\right\}$$

$$\left.+\frac{1}{2}\sum_{k=1}^{k_j}\left(\frac{-1}{\lambda}-0\right)\right]$$

$$\frac{\partial^2 l(\theta)}{\partial \lambda^2}=\sum_{J=1}^{j}\sum_{i=1}^{N_j}\left[\frac{Y_{ij^2}(t_{ij}k_j)}{2\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}+\frac{1}{2}\left\{\frac{2\Lambda^2(t_{ij}k_j)\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)-Y_{ij}(t_{ij}k_j)\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta\right)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2}\right\}\right.$$

$$-\left\{\frac{Y_{ij}(t_{ij}k_j)\Lambda(t_{ij}k_j)\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^2-2\left(\lambda\Lambda(t_{ij}k_j)+k^2\omega\right)\left(\lambda Y_{ij^2}(t_{ij}k_j)+Y_{ij}(t_{ij}k_j)+k^2\right)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)^4}\right\}$$

$$\left.+\frac{1}{2}\sum_{k=1}^{k_j}\left(-\frac{1}{\lambda^2}\right)\right] \quad \cdots (21)$$

$$\frac{\partial l(\theta)}{\partial \beta}=\sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[0+\frac{1}{2}\left\{\frac{2\left(\lambda\Lambda(t_{ij}k_j)+k^2\delta_j\right)}{\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)}\lambda\frac{\partial\Lambda_{ijk}}{\partial\beta}-0\right\}+\frac{1}{2}\sum_{k=1}^{k_j}\left\{\frac{1}{\lambda\Lambda_{ijk}^{\ 2}}2\lambda\Lambda_{ijk}\frac{\partial\Lambda_{ijk}}{\partial\beta}-\frac{2\lambda\Lambda_{ijk}}{Y_{ijk}}\frac{\partial\Lambda_{ijk}}{\partial\beta}\right\}\right]$$

$$\frac{\partial l(\theta)}{\partial \beta}=\sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left\{\frac{\left(\lambda\Lambda_{ijk}+k^2\delta_j\right)}{\left(\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)\right.}\frac{\partial\Lambda_{ijk}}{\partial\beta}\right\}+\frac{1}{2}\sum_{k=1}^{k_j}\left\{\frac{2}{\Lambda_{ijk}}\frac{\partial\Lambda_{ijk}}{\partial\beta}-\frac{2\lambda\Lambda_{ijk}}{Y_{ijk}}\frac{\partial\Lambda_{ijk}}{\partial\beta}\right\}\right]$$

$$\frac{\partial l(\theta)}{\partial \beta}=\sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left(\frac{\partial\Lambda_{ijk}}{\partial\beta}\left\{\frac{\lambda\left(\lambda\Lambda_{ijk}+k^2\delta_j\right)}{\left(\left(\lambda Y_{ij}(t_{ij}k_j)+k^2\right)\right.}\right\}\right)+\sum_{k=1}^{k_j}\left(\frac{1}{\Lambda_{ijk}}-\frac{2\lambda\Lambda_{ijk}}{Y_{ijk}}\right)\frac{\partial\Lambda_{ijk}}{\partial\beta}\right] \quad \cdots (22)$$

$$\frac{\partial^2 l(\theta)}{\partial \beta^{\,2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left\{\frac{\partial \Lambda_{ijk}}{\partial \beta}\frac{\lambda\left(\lambda\frac{\partial \Lambda_{ijk}}{\partial \beta}+0\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)}+\left(\frac{\lambda\left(\lambda \Lambda_{ijk}+k^2\delta_j\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)}\right)\right\} + \sum_{k=1}^{k_j}\left\{\frac{\partial \Lambda_{ijk}}{\partial \beta^2}\left(\frac{1}{\Lambda_{ijk}}-\frac{2\lambda \Lambda_{ijk}}{Y_{ijk}}\right)+\left(-\frac{1}{\Lambda_{ijk^2}}\frac{\partial \Lambda_{ijk}}{\partial \beta}-\frac{2\lambda}{Y_{ijk}}\frac{\partial \Lambda_{ijk}}{\partial \beta}\right)\frac{\partial \Lambda_{ijk}}{\partial \beta}\right\}\right]$$

$$\frac{\partial^2 l(\theta)}{\partial \beta^{\,2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left\{\frac{\lambda^2}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)}\left(\frac{\partial \Lambda_{ijk}}{\partial \beta}\right)^2+\left(\frac{\lambda\left(\lambda \Lambda_{ijk}+k^2\delta_j\right)}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)}\right)\frac{\partial^2 \Lambda_{ijk}}{\partial \beta^2}\right\} + \sum_{k=1}^{k_j}\left\{\frac{\partial^2 \Lambda_{ijk}}{\partial \beta^2}\left(\frac{1}{\Lambda_{ijk}}-\frac{2\lambda \Lambda_{ijk}}{Y_{ijk}}\right)+\left(-\frac{1}{\Lambda_{ijk^2}}-\frac{2\lambda}{Y_{ijk}}\right)\left(\frac{\partial \Lambda_{ijk}}{\partial \beta}\right)^2\right\}\right]$$

$$\frac{\partial^2 l(\theta)}{\partial \beta^{\,2}} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\left(\frac{\lambda\left(\frac{\partial \Lambda\left(t_{ij}k_j\right)}{\partial \beta}\right)^2+\left(\frac{\partial^2 \Lambda\left(t_{ij}k_j\right)}{\partial \beta^2}\right)\lambda \Lambda\left(t_{ij}k_j\right)+k^2\delta}{\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2}\right) + \sum_{k=1}^{k_j}\left\{\left(\frac{\partial^2 \Lambda_{ijk}}{\partial \beta^2}\right)^2\left(-\frac{1}{{\Lambda_{ijk}}^2}-\frac{2\lambda}{Y_{ijk}}\right) + \frac{\partial^2 \Lambda_{ijk}}{\partial \beta^2}\left(-\frac{1}{\Lambda_{ijk^2}}-\frac{2\lambda \Lambda_{ijk}}{Y_{ijk}}\right)\right\}\right] \quad (23)$$

$$\frac{\partial^2 l(\theta)}{\partial k \partial \beta} = \sum_{J=1}^{J}\sum_{i=1}^{N_j}\left[\frac{\left(\lambda\frac{\partial \Lambda\left(t_{ij}k_j\right)}{\partial \beta}\right)\left\{\left(2k\delta\lambda Y_{ij}\left(t_{ij}k_j\right)\right)+2k\delta_j-k\lambda \Lambda\left(t_{ij}k_j\right)+k^3\delta\right\}}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)^2}-\frac{k\lambda\frac{\partial \Lambda\left(t_{ij}k_j\right)}{\partial \beta}}{\left(\lambda Y_{ij}\left(t_{ij}k_j\right)+k^2\right)}\right] \quad \cdots (24)$$

$$\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1} = \sum_{j=1}^{J}\left[Z_j exp\left(\propto_0+\propto_1 x_j\right)\frac{\partial l(\theta)}{\partial \delta_j}+exp\left(\propto_0+\propto_1 x_j\right)\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}x_j\right]$$

$$\frac{\partial^2 l(\theta)}{\partial {\propto_0}^2} = \sum_{j=1}^{J} Z_j exp\left(\propto_0+\propto_1 Z_j\right)\left(\frac{\partial l(\theta)}{\partial \delta_j}+\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}\right) \quad \cdots (25)$$

$$\frac{\partial^2 l(\theta)}{\partial {\propto_0}^2} = \sum_{j=1}^{J}\left[exp\left(\propto_0+\propto_1 Z_j\right)\frac{\partial l(\theta)}{\partial \delta_j}+exp\left(\propto_0+\propto_1 Z_j\right)\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}\right]$$

$$\frac{\partial^2 l(\theta)}{\partial {\propto_0}^2} = \sum_{j=1}^{J} Z_j exp\left(\propto_0+\propto_1 Z_j\right)\left[\frac{\partial l(\theta)}{\partial \delta_j}+\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}\right] \quad \cdots (26)$$

$$\frac{\partial^2 l(\theta)}{\partial {\propto_1}^2} = \sum_{j=1}^{J}\left[Z_j exp\left(\propto_0+\propto_1 Z_j\right)\frac{\partial l(\theta)}{\partial \delta_j}+exp\left(\propto_0+\propto_1 Z_j\right)\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}\right]$$

$$\frac{\partial^2 l(\theta)}{\partial {\propto_1}^2} = \sum_{j=1}^{J} {Z_j}^2 exp\left(\propto_0+\propto_1 Z_j\right)\left[\frac{\partial l(\theta)}{\partial \delta_j}+\frac{\partial^2 l(\theta)}{\partial {\delta_j}^2}\right] \quad \cdots (27)$$

$$\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \beta} = \sum_{j=1}^{J}\left[exp\left(\propto_0+\propto_1 x_j\right) . \frac{\partial^2 l(\theta)}{\partial \delta_j \partial \beta}\right] \quad \cdots (28)$$

$$\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial\beta} = \sum_{j=1}^{J}\left[Z_j exp(\propto_0 + \propto_1 x_j) \cdot \frac{\partial^2 l(\theta)}{\partial \delta_j \partial\beta}\right] \quad \cdots (29)$$

$$\frac{\partial^2 l(\theta)}{\partial\lambda\partial\beta} = \sum_{j=1}^{J}\sum_{j=1}^{N_j}\left[\frac{1}{2}\left\{\frac{2\left(2\lambda\Lambda(t_{ij}k_j)\right)\frac{\partial\Lambda(t_{ij}k_j)}{\partial\beta} + k^2\delta\frac{\partial\Lambda(t_{ij}k_j)}{\partial\beta}}{\lambda Y_{ij}(t_{ij}k_j) + k^2} - \frac{Y_{ij}(t_{ij}k_j)\lambda\frac{\partial\Lambda(t_{ij}k_j)}{\partial\beta}}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^2}\right\} + \frac{1}{2}\sum_{k=1}^{k_j}\left(-\frac{\Lambda_{ijk}}{Y_{ijk}}\frac{\partial\Lambda_{ijk}}{\partial\beta}\right)\right] \quad \cdots (30)$$

$$\frac{\partial^2 l(\theta)}{\partial\lambda\partial\delta_j}\sum_{j=1}^{J}\sum_{j=1}^{N_j}\left[\frac{1}{2}\left(\frac{2\Lambda(t_{ij}k_jk^2)}{\lambda Y_{ij}(t_{ij}k_j) + k^2} - \frac{Y_{ij}(t_{ij}k_j)k^2}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^2}\right)\right] \quad \cdots (31)$$

$$\frac{\partial^2 l(\theta)}{\partial\lambda\partial \propto_0} = \sum_{j=1}^{J}\left[exp(\propto_0 + \propto_1 x_j) \cdot \frac{\partial^2 l(\theta)}{\partial\lambda\partial\delta_j}\right] \quad \cdots (32)$$

$$\frac{\partial^2 l(\theta)}{\partial\lambda\partial \propto_1} = \sum_{j=1}^{J}\left[Z_j exp(\propto_0 + \propto_1 Z_j) \cdot \frac{\partial^2 l(\theta)}{\partial\lambda\partial\delta_j}\right] \quad \cdots (33)$$

$$\frac{\partial^2 l(\theta)}{\partial k\partial\delta_j} = \sum_{j=1}^{J}\sum_{j=1}^{N_j}\left[\frac{1}{2}\frac{\left(2k\lambda^2\Lambda(t_{ij}k_j)Y_{ij}(t_{ij}k_j) + 4k^3\omega_j\lambda Y_{ij}(t_{ij}k_j) + k^3\right)}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^2} - 2k\delta_j\right] \quad \cdots (34)$$

$$\frac{\partial^2 l(\theta)}{\partial k\partial\lambda} = \sum_{j=1}^{J}\sum_{j=1}^{N_j}\left[\frac{\left(\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)Y_{ij}(t_{ij}k_j) - \lambda Y_{ij}(t_{ij}k_j)^2\right)}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^2} + \left(\frac{\left(\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^4\left\{4k^3\omega_j\lambda\Lambda(t_{ij}k_j) + 2k^3\delta_j{}^2Y_{ij}(t_{ij}k_j)k\Lambda(t_{ij}k_j)\right\}\right)}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^4}\right) - \left(\frac{\left(\left\{(\lambda\Lambda(t_{ij}k_j) + k^2\omega)2k\delta_j\lambda Y_{ij}(t_{ij}k_j) + k^3\delta_j - k\lambda\Lambda(t_{ij}k_j)\right\}\left(2\lambda Y_{ij}(t_{ij}k_j)^2 + 2Y_{ij}(t_{ij}k_j) + k^2\right)\right)}{\left(\lambda Y_{ij}(t_{ij}k_j) + k^2\right)^4}\right)\right] \quad \cdots (35)$$

$$\frac{\partial^2 l(\theta)}{\partial k\partial \propto_0} = \sum_{j=1}^{J} exp(\propto_0 + \propto_1 Z_j)\frac{\partial^2 l(\theta)}{\partial k\partial\delta_j} \quad \cdots (36)$$

$$\frac{\partial^2 l(\theta)}{\partial k\partial \propto_1} = \sum_{j=1}^{J} Z_j exp(\propto_0 + \propto_1 Z_j)\frac{\partial^2 l(\theta)}{\partial k\partial\delta_j} \quad \cdots (37)$$

$$\frac{\partial l(\theta)}{\partial \propto_0} = \sum_{j=1}^{J} exp(\propto_0 + \propto_1 Z_j)\frac{\partial l(\theta)}{\partial\delta_j} \quad \cdots (38)$$

$$\frac{\partial l(\theta)}{\partial \propto_1} = \sum_{j=1}^{J} Z_j exp(\propto_0 + \propto_1 Z_j)\frac{\partial l(\theta)}{\partial\delta_j} \quad \cdots (39)$$

and then the fisher information matrix can be developed as given below:

$$
\begin{matrix}
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0{}^2}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1{}^2}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k^2}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \lambda^2}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \lambda \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \beta}\right] & \left[-\frac{\partial^2 l(\theta)}{\partial \lambda \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \beta^2}\right]
\end{matrix} \quad \cdots (40)
$$

The log-likelihood function can be maximized to obtain maximum likelihood estimator MLEs. The direct maximization of log-likelihood function gives equations which are computationally difficult to solve. Under the truncated normal distribution, direct maximization of the likelihood function often yields a solution far away from the MLE.

Numerical Example

Following the approach of Yang et.al.(2007)is used here to illustrate the proposed procedure. A case study was observed in the Micro Electro Mechanical System Lab(MEMS LAB), Faculty of Engineering and technology, Annamalai university, a total of 50 resistors in a constant stress ADT, Where 30 Samples observed at the electrical connector is failed if the data are collected under three temperature levels: 55℃, 75℃, 100℃ .where observed every measurement time stress different temperature, it was assume that the normal use temperature and threshold value for percent increase in resistance was l=6, the samples is tabulated in table 1,the 7th point of the second unit under 55℃ labeled blank as indicated by Yang et al.(2007) to preserve the monotone behaviour of the stress relaxation and the measurement approaches under each temperature level. For more detailed discussion refer toBagdonavicius and Nikulin, (2001), Lawless, and Crowder, (2010), Leydold and Hörmann, (2011).

Table 1 Stress relaxation data under the temperature level

Temperature	Sl.No	Stress loss	Mean Time
55℃	1	2.13, 2.06, 3.43, 4.36, 5.86, 6.24, 6.63, 7.34, 7.58, 8.42, 9.57	7.60
	2	2.34, 3.65, 4.69, 4.85, 5.36, 0, 6.59, 8.48, 9.35, 10.95	
	3	2.8, 3.56, 4.65, 5.89, 6.3, 7.65, 8.95, 9.21, 10.45, 11.32	
	4	2.96, 3.58, 5.38, 5.32, 7.68, 8.27, 8.61, 9.854, 10.97, 11.57	
	5	3.65, 4.55, 5.33, 7.58, 8.39, 9.37, 9.33, 10.24, 11.89, 12.54, 13.59	
	6	3.59, 5.69, 5.87, 6.29, 8.98, 10.25, 11.00, 12.69, 13.69, 15.91	
75℃	7	2.98, 4.98, 5.87, 6.38, 8.56, 10.21, 11.98, 11.00, 13.24, 15.38	10.65
	8	3.65, 4.27, 6.29, 8.91, 9.54, 10.14, 12.69, 14.32, 16.90	
	9	3.69, 4.28, 6.72, 8.34, 8.64, 10.81, 11.20, 14.57, 16.90, 18.18	
	10	3.58, 4.92, 6.91, 7.34, 9.38, 11.78, 12.98, 13.92, 15.39, 18.29	
	11	3.58, 4.87, 7.96, 8.64, 10.94, 12.61, 13.94, 15.38, 17.82, 19.34	
	12	5.96, 5.89, 8.91, 9.67, 12.67, 13.54, 15.98, 17.51, 20.64, 23.94	
100℃	13	4.89, 5.91, 8.47, 9.38, 11.84, 13.57, 15.94, 16.97, 18.54, 19.82	14.09
	14	4.94, 6.85, 7.95, 9.64, 10.87, 12.67, 15.47, 16.32, 18.94, 21.98	
	15	5.97, 6.31, 8.57, 10.91, 12.97, 14.51, 16.78, 18.96, 19.49, 21.34	
	16	4.25, 7.58, 9.34, 10.64, 13.95, 15.27, 16.97, 19.84, 20.46, 22.7	
	17	5.94, 6.28, 8.94, 12.73, 14.61, 16.37, 18.39, 21.78, 22.96, 24.75	
	18	4.18, 8.91, 10.94, 12.71, 15.67, 17.64, 19.78, 21.64, 24.97, 28.45	

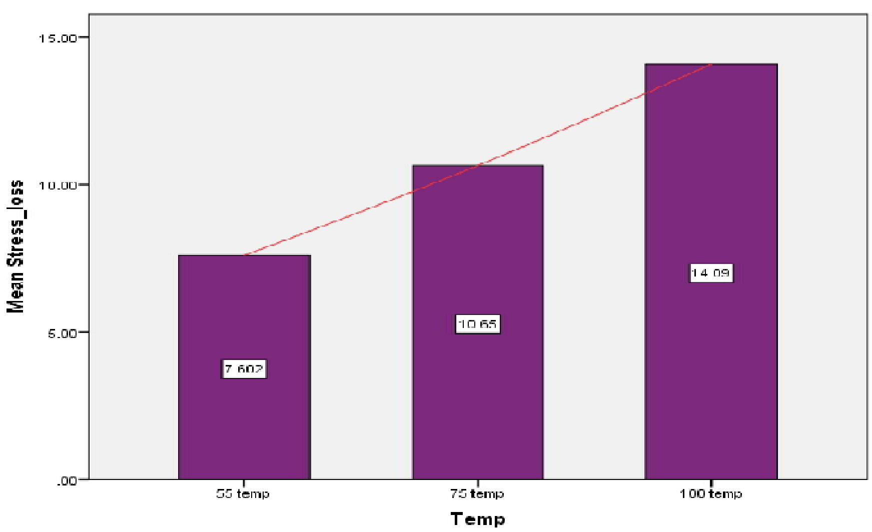

Table 2 Measurement time under differenttemperatures

Temperature	Measurement time epochs (in hours)
55℃	107,238, 540, 838, 1063, 1249, 1536, 1789, 2164, 2414, 1812
75℃	45, 109, 247, 411, 641, 758, 1017, 1232, 1621, 249
100℃	44, 110, 204, 322, 457, 684, 847, 1041, 1204

In the following, we will determine the optimal ADT plans based on both models. Assume 10 units are available for the ADT test. In the ADT, we set $\tau_j = 24$, and $k_j = 14$ for all $j = 1,2,\dots.J$.this setting means that we measure the degradation level once every day, and the test lasts two weeks. Our planning involves selecting the stress level, $(x_1, x_2, \dots, x_{J-1})$, and the proportion of samples allocated to each testing level, $(N_1, N_2, \dots, N_{J-1}))$. Consider a two-level ADT plan. Suppose we are interested in minimizing the asymptotic variance of 10, the 0.1-quantile of the failure time distribution at use conditions. When,$J = 2$ yields the optimal ADT design

The elements of fisher matrix by solving through mat lab are:

$$\begin{bmatrix} -1.258\times10^{8} & -1.269\times10^{8} & -1.6891\times10^{9} & -20.91\times10^{8} & -1.62\times10^{5} \\ -1.18\times10^{8} & -8.94510\times10^{7} & -1.6541\times10^{9} & -15.7351\times10^{8} & -1.127\times10^{8} \\ -1.26578\times10^{9} & -1.3298\times10^{9} & -6.791\times10^{9} & -8.734\times10^{12} & -1.339\times10^{9} \\ -21.32\times10^{8} & -15.761\times10^{8} & -8.458\times10^{12} & -4.9780\times10^{9} & -1.38\times10^{7} \\ -1.29\times10^{5} & -1.113\times10^{8} & -1.325\times10^{8} & -1.39\times10^{7} & -5.69\times10^{7} \end{bmatrix}$$

Table 3 Optimization table for random drift model

Process	x_1	x_2	N_1	N_2	$Std(\varphi p)$
Random drift model	0	1	1	9	4216

The optimal ADT design is shown in the above table. It is attractive to observe that optimal lower stress value is 0. This result is true because the degradation under the normal use condition is quick enough so that the error caused by extrapolation to the failure threshold is small, even if we test the unit under use conditions.

Table 4 Optimization table for simple IG process

Process	x_1	x_2	N_1	N_2	$Std(\varphi p)$
Simple Inverse Gaussianmodel	0	1	1	9	17450

SUMMARY AND CONCLUSION

In this Paper we have considered random drift model for the study since this model takes into result unit to unit variation of the sample of product. Different methods are developed with the time to test the product. But in the electronic industry accelerated degradation test gets more usefulness compared to the other methods. Since company produces large sample of similar products so there is need to test the product in short duration. So accelerated degradation test is more suitable and effective for studying the degradation performance since in life testing, increase the value of stress to fail the part quickly and collect the degradation data for predicting the reliability of the product.

Different type of accelerating degradation models have developed with the time and can be used in different types of situations. But, it has become necessary for the manager to test how many no of units should be tested at a particular stress level so that the cost of testing is less. Simple Stress Accelerated Degradation Test method has been developed by considering various criterions required such as robustness of design, optimality of design, tightened the value of constrained etc. So, inverse Gaussian process is used for the optimization of no of units and stress value. The proposed model discussed in this paper provides estimation of the no of units necessary for optimum stress level by minimizing the value of asymptotic variance. Fisher information matrix is a useful tool for estimating value of vectors used for finding the asymptotic variance

Reference

1. Bagdonavicius, V., and Nikulin, M. S. (2001). Estimation in degradation models with explanatory variables. *Lifetime Data Analysis*, 7(1), 85-103.
2. Folks, J. L., and Chhikara, R. S. (1978). The inverse Gaussian distribution and its statistical application--a review. *Journal of the Royal Statistical Society*. Series B (Methodological), 263-289.
3. Lawless, J. F., and Crowder, M. J. (2010). Models and estimation for systems with recurrent events and usage processes. Lifetime data analysis, IEEE. 16(4), 547-570
4. Chhikara, R.S., and Folks, J.L. (1989). The inverse gaussian distribution. Statistics: textbook and monographs.
5. Tweedie M.C.K., (1957). Statistical properties of inverse Gaussian distributions I, II, Annals of Mathematical Statistics 28, 362-377.
6. Balakrishna, N., Rahul, T. (2014). Inverse Gaussian distribution for Modelling Conditional Durations in Finance. Communications in Statistics - Simulation and Computation, 43:3, 476-486.
7. Leydold, J. Hörmann, W. (2011). Generating generalized inverse Gaussian random variates by fast inversion. Computational Statistics and Data Analysis, 55, 213-217.

8. Y Yang, G., and Yang, K. (2002). Accelerated degradation-tests with tightened critical values. Reliability, IEEE Transactions on, 51(4), 463-468.
9. G Yang,(2007) Life cycle Reliability Engineering In Hoboken, NJ, Wiley, USA.

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