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Research Article

CRITICAL GRAPHS OF S-VALUED GRAPHS

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A B S T R A C T

In [4], the authors introduced the notion of semi ring valued graphs. In [3], the authors introduced the notion of regularity on S- Valued graphs. In [5], we have introduced the notion of coloring on S-valued graphs. In [6], we have introduced the notion of K-coloring on S-valued graphs. In this paper, we study the critical graphs of S-valued graphs.

INTRODUCTION

The problem of coloring of a graph is equivalent to the problems of partitioning the vertex set into subsets, where each subset consist of vertices of the same color. This problem of colorings finds its application in storage of chemicals, or matching problems, scheduling problems. The problem of colorings in crisp graph is dealt in [2] by Jenson. In [4] the authors introduced the notion of semi ring valued graphs. In [5] we have introduced the notion of colorings on S-valued graphs. In [6] we have introduced the notion of k-colorable S-valued graphs. In [7] we have introduced the notion of chromatic number of some S-valued graphs. In this paper we study the concept critical graphs in some S-valued graphs.

Preliminaries

In this section, we recall some basic definitions that are required for our work.

Definition [1]: A semi ring (S, +,·) is an algebraic system with a non-empty set S together with two binary operators + and · such that

1. (S, +, 0) is a monoid.
2. (S,·) is a semi group.
3. For all a, b, c ∈ S, a · (b + c) =a · b + a · c and (a + b) · c = a · c + b · c
4. 0 · x = x · 0 = 0 for all x ∈ S.

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Definition [1]: Let (S, +,·) be a semiring.' $\preceq$ ' is said to be a canonical preorder if for a, b ∈ S, a $\preceq$ b if and if there exists c ∈ S such that a + c = b

Definition [2]: A k – vertex colorings of a graph G is an assignment of k – colors to the vertices of G such that no two adjacent vertices receive the same color.

Definition [2]: A graph G that required k – different colors for its colorings and not less number of colors is called a k – chromatic graph and the number k is called the chromatic number of G, denoted by $\chi(G)$. That is $\chi(G)$ = k.

***Definition*:** A graph G is called critical if $\chi(H) < \chi(G)$ for every proper subgraph H of G.

***Definition**:* A Graph G is called k- critical if $\chi(G)$=k and for each v∈V(G) , $\chi(G-v)$< χ(G)

Definition [3]: Let G = (V, E) be a given graph with V, E ≠ϕ. For any semi ring (S,+,·) a Semi ring - valued graph (or S-valued graph) G^S is defined to be the graph G^S = (V, E, σ, ψ) where σ : V → S andψ: E → S is defined to be

$$\psi(x,y) = \begin{cases} \min\{\sigma(x),\sigma(y)\} & if\ \sigma(x) \preccurlyeq \sigma(y) or\ \sigma(y) \preccurlyeq \sigma(x) \\ 0 & otherwise \end{cases}$$

for every unordered pair (x, y) of E ⊆ V ×V. We call σ, a S-vertex set and ψ a S-edge set of the S-valued graph G^S.

Definition [5]: Consider the S-valued graph G^S = (V, E, σ, ψ). A colouring of G^S is given by a function f: V ×V→ S×C such that for all v∈V, f(v,v) = (σ(v), c(v)), c(v) ∈C.

Definition [5]: A coloring f: V×V→S × C is said to be proper weight-uni coloring, if ∀ v ∈V and c(v) ∈ C is the same, but σ(v) ∈ S differ for adjacent vertices.

Definition [5]: Consider a S-valued graph G^S. A coloring f on G^S is said to be equi-weight (or vertex regular) proper coloring if for all v ∈V, σ (v) have equal value in S and c(v) ∈ C differ for adjacent vertices.

Definition [5]: Consider a S-valued graph G^S. A coloring f on G^S is said to be total proper coloring if for all v ∈V,σ (v) ∈ S and c(v) ∈ C differ for adjacent vertices.

Definition [6]: Let G^S be a S-valued graph. The vertex chromatic number of G^S, denoted by $\chi_S(G^S)$, is defined to be $\chi_S(G^S) = (\min_{v\in V}\sigma(v), \min |C|)$.

Definition [6]: A S-valued graph G^S is said to be k-colorable, if it has a proper vertex regular or total proper colorings such that $|C| = k$.

Definitions [4]: T he degree of the vertex v_i of the S-valued graph G^S is defined as

G^S is defined as $\deg_S(v_i) = \sum_{(vi,vj)\in E}\psi(vi, vj), l$ where l is the number of edges incident with v_i.

Critical Graphs of S-Valued Graphs

In this section, we introduced the concepts of critical S-valued graphs.

Definitions: Consider a S-valued graphs $G^S = (V,E,\sigma,\psi)$ the graph G^S is said to be critical if $\chi_S(H^S) \lessdot \chi_S(G^S)$ for every proper S- valued sub graph H^S of G^S.

Let $G^S = (V,E,\sigma,\psi)$ be such that $\chi_S(G^S)=k$.

From G^S if we remove any vertex v∊V then the graph $H^S= G^S-\{(v.\sigma(v))\}$ is proper sub graph of G^S and also $\chi_S(H^S)= (\min_{u\in V-\{v\}}(\sigma(v), k-1) \lessdot \chi_S(G^S)$ This leads to the following definition.

Definitions: consider the S-valued graph $G^S=(V,E,\sigma,\psi)$ such that $\chi_S(G^S)= (\min_{v\in V}\sigma(v),k)$,G^S is said to be k-critical if $\chi_S(H^S)=\chi_S(G^S-\{(v,\sigma(v)\}) \lessdot \chi_S(G^S)$ for each vertex $(v,\sigma(v))$∈G^S.

In other words a critical graph G^S such that $\chi_S(G^S)=k$ is called a k-critical S- valued graph

Lemma: Every S-valued graph G^S with $\chi_S(G^S)= (\min_{v\in V}\sigma(v),k)$ is a k-critical S-valued graphs.

Proof: Consider the S-valued graphs $G^S = (V,E,\sigma,\boldsymbol{\psi})$ **let** $\boldsymbol{\chi_S(G^S)}= (\min_{v\in V}\sigma(v),k)$.

From G^S, if we remove any vertex then the S-valued graph H^S: $G^S-\{v.\sigma(v)\}$ is a proper subgraph of G^S . we determine $\chi_S(H^S)$ .If $\chi_S(H^S)$ is not less than or equal to $\chi_S(G^S)$, we remove a vertex from H^S to get a proper subgraph H_1^S of H^S which is again a proper subgraph of G^s. We determine $\boldsymbol{\chi_S(H_1^S)}$. If $\boldsymbol{\chi_S(H_1^S)}$ is not less than or equal to $\chi_S(G^S)$, we remove one vertex from $\mathbf{H_1^S}$.

We continue the process until the chromatic number of the proper subgraph can not further be reduced. The final subgraph $\mathbf{H_k^S}$ obtained after a finite number of steps, say, k such that $\boldsymbol{\chi_S(H_k^S)} \lessdot \chi_S(G^S)$, proves that G^S is a k-critical S-valued graphs.

Theorem: Any k-critical S-valued graph G^S has minimum degree at least $(\min_{u\in V}\sum_{v\in N_S(u)}\psi(u,v), k-1)$

Proof: Assume that there exit a vertex v in G^S such that $\deg_S(v)\le(\sigma(v),k-2)$ Therefore $|N_S(v)|=k-2$ thus there are k-2 vertices. in the neighborhood v which can be colored by using at most k-2 colors, Consider the sub graph $H^S=G^S-\{(v,\sigma(v)\}$ which is a proper subgraph of G^S .Since G^S is k-critical, $\chi_S(H^S)= \min_{u\in V-\{v\}}(\sigma(v), k-1)$. This implies that at least one colors is available for the vertex v. Thus G^S can be properly colored with k-1 colors. Contradicting

$$\chi_S(G^S) = (\min_{u\in V}\sum_{v\in N_S(u)}\psi(u,v), k)$$

Hence there exits vertex v∈ V with atleast degree $(\min_{u\in V}\sum_{v\in N_S(u)}\psi(u,v), k-1)$.

Theroam: Every k-critical S-valued graph G^S is S-connected

Proof: Suppose G^S is not S- Connected and $\chi_S(G^S) = (\min_{v\in V(G)}\sigma(v), k)$.

Then there is a component G_1^S of G^S such that $\chi_S(G_1^S) = (\min_{v\in V(G_1)}\sigma(v), k)$

Let v be the vertex in G^S such that v does not belongs to $V(G_1)$. Then G_1^S is a component of the S-valued sub graph $G^S-[[v.\sigma(v)]]$.

Therefore $\chi_s(G^S-[v,\sigma(v)]=$
$\chi_S(G_1^S) = (\min_{v\in V(G_1)}\sigma(v), k)$. This contradicts, the definition G^S is a k- critical S- valued graphs, proving that G^S is connected.

Theorem : Every critical S-valued graph G^S is S- connected.
Proof : Let G^S be a critical S-valued graph, there fore $\chi_S(H^S) \lessdot \boldsymbol{\chi}_S(G^S)$ for evry proper of S-subgraph of G^S.

To prove: G^S is S-connected. Suppose not , Let $\chi_S(G^S) = (\min_{v\in V(G)}\sigma(v), k)$ there is a component G_1^S of G^S such that $\chi_S(G_1^S) = (\min_{v\in V(G_1)}\sigma(v), k)$

Let v be the vertex in G^S such that v does not belongs to $V(G_1)$. Then G_1^S is a component of S- the sub graph $G^S-[[v.\sigma(v)]]$ therefore $\chi_s(G^S-[v,\sigma(v)]=\boldsymbol{\chi}_s(G_1^S)= (\min_{v\in G_1}\sigma(v), k)$. This contradicts the definition that G^S is a critical S- Valued graphs, proving G^S is S-connected.

Theorem: Every S-connected k-chromatic graph contains a critical k- chromatic S-valued graph.

Proof : Let G^S be S-connected k-chromatic graph. Then

$\chi_S(G^S) = (\min_{v\in V(G)}\sigma(v), k)$

If G^S is not k-critical then $\chi_S(G^S-(v,\sigma(v)) = (\min_{u\in V-\{v\}}\sigma(u), k)$ for some vertex v of G^S.If $G^S-(v,\sigma(v))$ is k critical, then it is the required sub graph,if not then

$G^S-\{(v,\sigma(v)),(w,\sigma(w))\}=(G^S-(v.\sigma(v)))-(w,\sigma(w))$
has a chromatic number k. for some vertex w in G^S.$-(v,\sigma(v))$. That is

$\chi_S(G^S$-$\{(v,\sigma(v)).(w,\sigma(w))\}=\chi_S[(G^S$-$(v,\sigma(v))$-$(w,\sigma(w))] = (\min_{u\in V-\{v,w\}} \sigma(u), k)$

If this new S-subgraph is k-critical . then again it is the required sub graph, if not we continue this vertex deletion procedure untill we get a k-critical S-valued sub graph.

Theorem: Every odd Cycle $C^S_{\{2n+1\}}, n \geq 1$ is the only 3 critical S- valued graph

Proof: **L**et C_q^S be a cycle. We prove this theorem by induction on q. Let q=3 start with any vertex say v_1 in C_q^S,assign colour c_1 to v_1 .Then consider $N_S[V_1]$. If $v_2\epsilon N_S[V_1]$ it should be assigned a colour different from c_1. Let v_2 be assigned c_2.. If $v_3\epsilon N_S[V_1]\cap N_S[V_2]$, v_3 should be assigned c_3 different from $c_1$and c_2. Thus in $C_3^S=K_3^S$, $\chi_S(C_3^S)=3$, clearly $\chi_S(C_3^S$-$(v,\sigma(v))=2$ it is not a cycle ,. Hence C_q^S is a 3- critical , let us assume the above theorem is holds good for Ct^S . That is C_t^S is t- critical, t is odd.

Claim: C_{t+2}^S is 3- critical.

Let $v_1\epsilon\ C_{t+2}^S$ If $v_1\varepsilon\ C_t^S$ then it must be assigned by any one of c_t colour say c_1 then consider $N_S[V_1]$. If $v_2\epsilon N_S[V_1]$ and $v_2\notin C_t^S$ then v_2 must have a colour . say c_{t-1}. Since it is of odd cycles.>c_t it contains atleast 2 edges. Both incident at v_2. Therefore there is another vertex v_3 which is adjacent to v_2 and the vertex in C_t^S .Therefore v_3 must assigned a colour c_1 $\Longrightarrow \chi_S(C_{t+2}^S)= 3$.,Clearly $\chi_S(C_{t+2}^S$ –$(v.\sigma(v))= 2$, is not a cycle. Therefore C_{t+2}^S is 3- critical . Hence by induction C_{t+2}^S is 3 critical for n≥1.

CONCLUSION

In this paper, we have discussed the critical graphs for S-valued graphs. Further investigation will be done on colouring of graph products and the critical graphs of S-valued graphs.

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