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Research Article

MODELLING AND ANALYSIS OF A PREDATOR-PREY SYSTEM UNDER HERD-BEHAVIOR OF PREY

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A R T I C L E I N F O

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A B S T R A C T

A prey-predator model is considered in which Lotka-Voltera interaction term is used as the functional response of the predator to the prey population. The interaction term is proportional to the square root of the prey population in which prey exhibits herd structure. Here we consider that both prey and predator have alternative growth rate. We check the behavior of the different equilibrium points of the model both analytically and numerically.

INTRODUCTION

It is well known that the population dynamics of different species are inevitably linked. In recent years researchers have taken several approaches to study this interesting phenomenon in prey predator system. Previously a good number of studies have shown that predators take disproportionate number of prey that are infected by parasites (see, Vaughn and Coble [1]; Temple [2]). Chattopadhyay and Arino [8] studied predator-prey system when predator eat infected prey and derived the persistence and extinction conditions and also determined the condition for Hopf bifurcation. Xiao and Chen [11] modified the model of Chattopadhyay and Arino by introducing the delay term and studied the dynamics of the modified system. Mukherjee [16] analysed a generalized prey-predator system with parasite infection and obtained conditions for persistence and impermanence. Roy and Chattopadhyay [10] introduced a mathematical model of disease-selective predation incorporating this concept. They considered a predator-prey system where the predator has specific choice regarding predation and it can recognize the infected prey and avoid those during predation. Holmes and Bethel [3] discussed many examples in which the parasite changes the external features or behavior of the prey, so that infected prey are more vulnerable to predator. Infected prey sometimes choose such locations that are more accessible to predators; for example, infected fish or aquatic snails may live close to the water surface or snails may live on top of vegetation rather than under protective plant cover. Similarly, infected prey sometimes became weaker or less active, so that they are caught more easily by predator (see [22]). In a prey-predator model with disease in prey Anderson and May [5] found that the pathogen tends to destabilize the prey-predator interactions and exhibits limit cycles when predation on infected prey is much and no reproduction in infected prey. Here we consider a prey predator model where both prey and predator species have alternative growth and prey are in group. That is they are in herd behavior. We firstly find out equilibrium points and stability analysis of these equilibrium points. Finally we draw the attention numerically to support the analytical result.

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Mathematical Model Formation

Here the following assumptions are made to formulate the mathematical model

1. We assume that prey population $x(t)$ grows logistically in the absence of predator with intrinsic growth rate r and carrying capacity k of the prey population.
2. Also the prey population gathered in a group to form a herd as their defensive mechanism to save themselves from predator population.
3. For predator population $y(t)$, it is assumed that there is a natural logistic growth with intrinsic growth rate r_1 and carrying capacity k_1.

4. It is also assumed that, beside logistic growth the predator population has an alternative food source which they get by consuming the prey population.

Based on the above mentioned assumption we consider the model as:

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \beta\sqrt{x}y\,,$$
$$\frac{dy}{dt} = r_1 y\left(1-\frac{y}{k_1}\right) + \beta y\sqrt{x}. \qquad \ldots\ldots\ldots\ldots(1)$$

r,k,r_1, k_1, β are all positive constants.

Positive invariance of the system

Let us put equation (2.1) in a vector form by setting $x = col(x, y) \in R^2$

$$F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \end{bmatrix} = \begin{bmatrix} rx\left(1-\frac{x}{k}\right) - \beta\sqrt{x}y \\ r_1 y\left(1-\frac{y}{k_1}\right) + \beta y\sqrt{x} \end{bmatrix}.$$

Where $F\colon C_+ \to R^2$ and $F \in C^\infty$. The equation (2.2) becomes $\dot{X} = F(X)$, with $X(\theta) = (\emptyset_1(\theta), \emptyset_2(\theta)) \in C_+$ and $\emptyset_i(\theta) > 0\ (i = 1, 2)$. It is easy to cheek in the above equation that whenever choosing $X(\theta) \in C_+$ such that $X_i = 0$, then $F_i(x) : x_i(t) = 0,\ x(t) \in C_+ \geq 0, (i = 1, 2)$. Due to lemma (Yang et al.[18]) any solution of the above equation with $X(\theta) \in C_+$, say $X(t) = X(t, X(\theta))$, such that $X(\theta) \in R^2$ for all $t > 0$.

Boundedness of solution

Let us define $w = x + y$. The time derivative

$$\frac{dw}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = rx\left(1-\frac{x}{k}\right) - \beta\sqrt{x}y + r_1 y\left(1-\frac{y}{k_1}\right) + \beta y\sqrt{x}.$$

Now, $\frac{dw}{dt} + qw = rx\left(1-\frac{x}{k}\right) + r_1 y\left(1-\frac{y}{k_1}\right) + q(x+y)$

$$\frac{dw}{dt} + qw = k\frac{(r+q)^2}{4r} + k_1\frac{(r_1+q)^2}{4r_1} - \frac{r}{k}\left[x - \frac{k}{2r}(r+q)^2\right] - \frac{r_1}{k_1}\left[y - \frac{k_1}{2r_1}(r_1+q)^2\right] \leq B.$$

Where $B = k\frac{(r+q)^2}{4r} + k_1\frac{(r_1+q)^2}{4r_1}$, then $\frac{dw}{dt} + qw \leq B(constant)$, which is a linear differential equation in w. After solving we get, $w \leq \frac{B}{q} + Ce^{-qt}$ where C is an integrating constant. At $t = 0, w = 0$, so $C = -\frac{B}{q}$. Therefore $w \leq \frac{B}{q}(1 - e^{-qt})$ and since $w \geq 0$, so $0 \leq w \leq \frac{B}{q}(1 - e^{-qt})$. That is solutions of the system are all bounded.

Qualitative Analysis of the Model System

Equilibrium and Existence

The system of equation (2.1) has four equilibrium points, namely $E_0(0,0)$, $E_1(k,0)$, $E_2(0,k_1)$ and $E^*(x^*,y^*)$. The equilibrium point $E_0(0,0)$, $E_1(k,0)$ are unstable and the equilibrium point $E_2(0,k_1)$ is stable. The interior equilibrium point $E^*(x^*,y^*)$ can be obtained from the equation

$rx^*\left(1-\frac{x^*}{k}\right) - \beta\sqrt{x^*}y^* = 0$ and $r_1 y^*\left(1-\frac{y^*}{k_1}\right) + \beta y^*\sqrt{x^*} = 0$. The x^* can be obtained as $x^* = \frac{r_1^2}{\beta^2}(1-\frac{y^*}{k_1})^2$ and y^* can be obtained from the equation

$\frac{AB}{k_1^3}y^{*3} - \frac{3AB}{k_1^2}y^{*2} + \left(\frac{A(3B-1)}{k_1{}^3} + \beta\right)y^* - A(B-1) = 0.$ Here both of $A = r\frac{r_1}{\beta}$ and $B = \frac{r_1{}^2}{k\beta^2}$ are positive and so $\frac{AB}{k_1^3}$ and $\frac{3AB}{k_1^2}$ both are positive. Hence from the above equation we observe that there is at least one change of sign in the coefficient of the above equation. So unconditionally the above equation has at least one positive real root. Thus the interior equilibrium point $E^*(x^*,y^*)$ always exists.

Stability Analysis

We now discuss the stability of the interior equilibrium point $E^*(x^*,y^*)$. The variational matrix at this equilibrium point can be obtained as

$$J(x^*,y^*) = \begin{pmatrix} r - \frac{2rx^*}{k} - \frac{\beta y^*}{2\sqrt{x^*}} & -\beta\sqrt{x^*} \\ \frac{\beta y^*}{2\sqrt{x^*}} & r_1 - \frac{2r_1y^*}{k_1} + \beta\sqrt{x^*} \end{pmatrix}$$ and the corresponding characteristic equation is

$$\begin{vmatrix} r - \frac{2rx^*}{k} - \frac{\beta y^*}{2\sqrt{x^*}} - \lambda & -\beta\sqrt{x^*} \\ \frac{\beta y^*}{2\sqrt{x^*}} & r_1 - \frac{2r_1y^*}{k_1} + \beta\sqrt{x^*} - \lambda \end{vmatrix} = 0.$$ The characteristic roots are obtained as

$$2\lambda = r\left(1-\frac{2}{k}\right)x^* + \left(\frac{\beta}{2\sqrt{x^*}} - \frac{r_1}{k_1}\right)y^* \pm \sqrt{r\left(1-\frac{2}{k}\right)x^* + \left(\frac{\beta}{2\sqrt{x^*}} - \frac{r_1}{k_1}\right){y^*}^2 - 2\beta^2y^*}.$$

The sufficient condition for stability $r\left(1-\frac{2}{k}\right)x^* + \left(\frac{\beta}{2\sqrt{x^*}} - \frac{r_1}{k_1}\right)y^* < 0$ with $r\left(1-\frac{2}{k}\right)x^* + \left(\frac{\beta}{2\sqrt{x^*}} - \frac{r_1}{k_1}\right){y^*}^2 - 2\beta^2y^* < 0$.

Numerical Simulation and Conclution:

In this paper we show that the solution of the system are positive and attains bound.

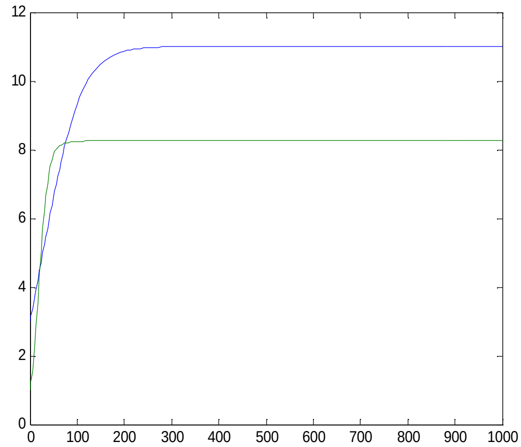

Figure 1(a) Shows that the stability of the interior equilibrium point taking the parametric values r=0.03,r_1=.1,k =12, k_1=8 and β= 0.001.

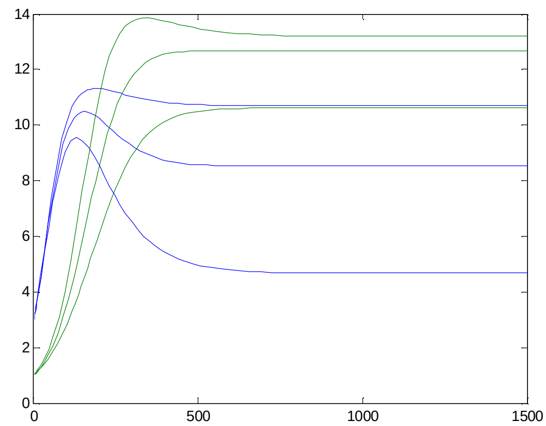

Figure 1b Shows that the stability of the interior equilibrium point taking the parametric values r=0.03,r_1=.01,k =12, k_1=8 and β= 0.001:0.001:0.003.

We find out all equilibrium point and find out the condition of existence of these equilibrium points. Then we show the stability of the model system. In this paper we try to point out how the prey gets advantage due to herd behavior and the existence and stability of the model when both the prey and predator species have their alternative growth rate. For understanding we try to solve the equation (1) numerically using Mat-lab. Firstly we find out the stability of the interior equilibrium point.

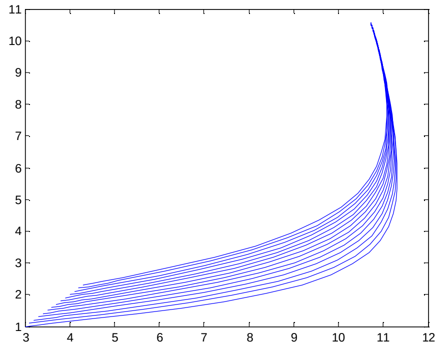

Figure 1 for β=0.001

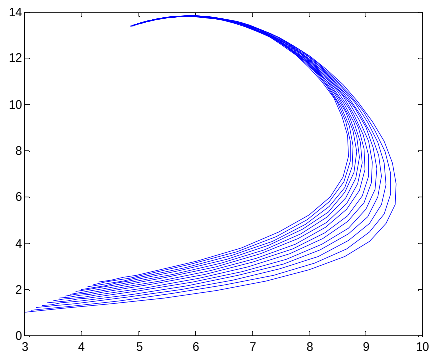

Figure 2 for β= 0.002

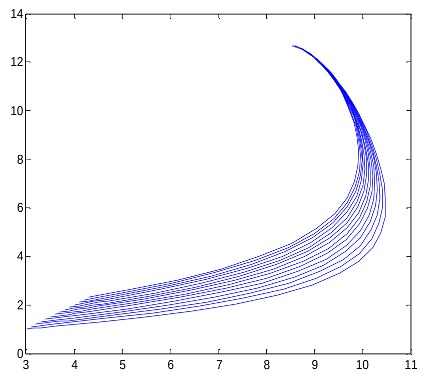

Figure 3 for β= 0.003

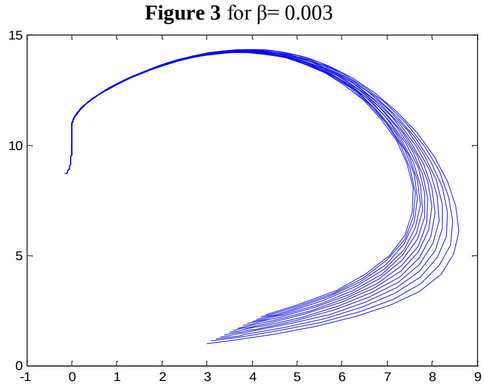

Figure 4 for β=0.004.

Figure (1) shows the surface plot of the model under taking the parametric values r=0.03, r_1=0.01, k=12, k_1=8 and β=0.001. Now we consider four different cases for neighbouring values of β and we find out four different surface plot. These figures show how the solution space changes for changing the values of β. Now keeping the parametric values same other than β we plot four figure as follows:

For figure (1) the solution of the system tends to (10.99, 9.11). For figure (2) the solution of the system tends to (9.19, 11.84). For figure (3) the solution of the system tends to (6.28, 13.78). For figure (4) the solution of the system tends to (2.63, 13.97). We show here if the rate of β increasing then how the solution of the model system changes.

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