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Research Article

CONSTRUCTION OF PARTIALLY EFFICIENCY BALANCED BLOCK DESIGNS WITH AND WITHOUT REPEATED BLOCKS

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A B S T R A C T

Two new methods have been proposed for the construction of Partially Efficiency Balanced Designs with and without repeated blocks. Method 1 discusses the construction of Partially Efficiency Balanced Design with repeated blocks by reinforcing some treatments in two BIBDs of the series $v_1 = b_1 = s, r_1 = k_1 = s-1,\ \lambda_1 = s-2$ and $v_2 = b_2 = s+1, r_2 = k_2 = s,\ \lambda_2 = s-2$. Method 2 discusses the construction of Partially Efficiency Balanced Design without repeated blocks by reinforcing some treatments in BIBDs of the series $v_1 = b_1 = s, r_1 = k_1 = s-1,\ \lambda_1 = s-2$ where s is any positive integer (≥ 3). The methods are illustrated with suitable example.

INTRODUCTION

Block designs are widely used in many fields of research. A wide range of Balanced Incomplete Block Design (BIBD) and Partially Balanced Incomplete Block Design (PBIBD) are available in literature. However, these are restricted to equi-replicate and equi-block sizes. A new class of incomplete block designs, termed as partially efficiency-balanced (PEB) design, was introduced by Puri and Nigam, 1975. These designs can be made available in varying replications and unequal block sizes.

In many comparative experiments, sometimes there may be a treatment (or the treatments), usually the control, which may be logically at a different position from the rest of the treatments. For instance, in plant breeding trials, generally limited material is available for the new strains as compared to standard (or check) varieties. Reinforced (Das, 1958), augmented (Federer, 1961) supplemented balanced (Pearce, 1960), nearly balanced (Nigam, 1976) and orthogonally supplemented balanced (Calinski, 1971) designs were all developed for such experimentation. The basic approach in all such designs is to augment (supplement) any standard (generally the randomised block or balanced incomplete block) design with a set of new treatments.

A block design $D(v, b, \underline{k}, \underline{r})$ is said to be efficiency-balanced (EB) design (Puri and Nigam,1975) if for all treatment contrasts $s'T$

$$M_0 s = \mu s,$$

where μ is the unique non-zero eigen value of M_0 with multiplicity $(v-1)$ where

$$M_0 = r^{-\delta} P - 1r'/N$$

and

$$P = nk^{-\delta} n'.$$

As a generation to EB designs, (Puri and Nigam, 1977) defined an $m-$efficiency class PEB design as follows.

A block design $D(v, b, \underline{k}, \underline{r})$ is said to be a PEB design with $m-$efficiency class if

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there a set of $(v-1)$ linearly independent contrasts $\{s_{ij}\}$ which can be partitioned into m (≤ v-1) disjoint classes such that all the ρ_i contrasts of i^{th} class are estimated with the same relative loss of information μ_i, i.e., they satisfy the equation

$$M_0 s_{ij} = \mu_i s_{ij}, i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, \rho_i$$

So that the efficiency factor associated with every contrast of the $i-th$ class is $(1-\mu_i)$, where μ_i are the eigen value of M_0 with multiplicities $\rho_i \left(\sum \rho_i = v-1\right)$.Here $M_0 = M - (1/n)\underline{1}\underline{r}'$, $M = R^{-1}P$, $\quad P = NK^{-1}N'$

where $R = diag\{r_1, r_2, ..., r_v\}$ and $K = diag\{k_1, k_2, ..., k_b\}$ are diagonal matrices with diagonal elements as usual replication vector $\underline{r} = (r_1, r_2, ..., r_v)'$, and as the vector $\underline{k} = (k_1, k_2, ..., k_b)'$ of block sizes, respectively, and n denotes the total number of experimental units with $\underline{1}$being a column vector of unity of an appropriate size.

In particular, if $\mu_i = \mu$ for all i, the design is called an EB design which was discussed by Calinski, 1971, Puri and Nigam,1975, Williams, 1975 and so on. Most works on PEB designs in the literature have been devoted to discussion of the construction and analysis of the designs. Various properties of parameters of PEB designs have been studied by Kageyama and Puri, 1985.

In this paper two new methods have been proposed for the construction of Partially Efficiency Balanced Designs with and without repeated blocks. Method 1 discusses the construction of Partially Efficiency Balanced Design with repeated blocks by reinforcing some treatments in two BIBDs of the series $v_1 = b_1 = s$, $r_1 = k_1 = s-1$, $\lambda_1 = s-2$ and $v_2 = b_2 = s+1$, $r_2 = k_2 = s$, $\lambda_2 = s-2$. Method 2 discusses the construction of Partially Efficiency Balanced Design without repeated blocks by reinforcing some treatments in BIBDs of the series $v_1 = b_1 = s$, $r_1 = k_1 = s-1$, $\lambda_1 = s-2$ where s is any positive integer (≥3). The methods are illustrated with suitable example.

METHODS OF CONSTRUCTION

The following theorem deals with the method of constructing PEB design with repeated blocks using the incidence matrices of two BIBD of the series $v_1 = b_1 = s, r_1 = k_1 = s-1, \lambda_1 = s-2$ and $v_2 = b_2 = s+1, r_2 = k_2 = s$, $\lambda_2 = s-2$.

Theorem 1

Consider a BIBD design D_1 with the parameters $v_1 = b_1 = s, r_1 = k_1 = s-1, \lambda_1 = s-2$. Again consider another BIBD design D_2 with parameters $v_2 = b_2 = s+1, r_2 = k_2 = s$, $\lambda_2 = s-2$, where s is any positive integer (≥3). Now define an incidence matrix N as

$$N = \begin{bmatrix} N_{1_{(v_1 \times b_1)}} & \vdots & & \vdots & \overbrace{I_{v_1} \cdots I_{v_1}}^{Repeated\ s-2\ times} \\ & \vdots & N_{2_{(v_2 \times b_2)}} & \vdots & \\ O_{1_{(1 \times b_1)}} & \vdots & & \vdots & \underbrace{1_{(1\times b_1)} \cdots 1_{(1\times b_1)}}_{Repeated\ s-2\ times} \end{bmatrix}_{(v_1+1)\times(b_1+b_2+s(s-2))v} \tag{1}$$

where I_{v_1} is the unit matrix repeated $(s-2)$ times. Here N_1 and N_2 are the incidence matrices of the designs D_1 and D_2 respectively; $1_{1 \times b1}$ is the unit vector of order $(1 \times b_1)$; $O_{1 \times b_1}$ is the null vector. Then the design with the incidence matrix N gives unequally replicated partially efficiency balanced block design with the following parameters.

$$v = v_1 + 1; \ b = b_1 + b_2 + s(s-2);$$

$\underline{r} = (r_1^*, r_2^*)$, where $r_1^* = (r_1 + r_2 + s - 2)$ and $r_2^* = (r_2 + s(s-2))$

$\underline{k} = \left(k_1 \underline{1}'_{b_1}, k_2 \underline{1}'_{b_2}, 2\underline{1}'_{s(s-2)}\right)$; $\lambda = \lambda_1 + \lambda_2$

Proof

The incidence matrix defined in (1) consists of $v_1 + 1$ row. Considering these rows as treatments, obviously, the numbers of treatments are $v = v_1 + 1$. Also it is obvious that, since the design D_1 has b_1 blocks, the design D_2 has b_2 blocks and the identity matrices of orders s which are repeated $(s-2)$ times and hence $b_1+b_2+s(s-2)$ columns. By considering these columns as the blocks, the numbers of blocks are $b=b_1+b_2+s(s-2)$.

In the incidence matrix (1) out of v_1+1 rows up to v_1 rows in design D_1 each treatments occurs r_1 times, in D_2 it occurs r_2 times and identity matrix is repeated $(s-2)$ times and hence the number of replication is $r_1^* = (r_1 + r_2 + s - 2)$. Again in the last row a treatment in D_2 occurs r_2 times and 1 occurs $s(s-2)$ times and hence numbers of replications for $(v_1+1)^{th}$ treatment is $r_2^* = (r_2 + s(s-2))$

Design D_1 has k_1 blocks and D_2 has k_2 blocks, so block sizes up to b_1 and b_2 blocks are k_1 and k_2 respectively. However for the remaining blocks, block size is two. Hence the block size of the design D is $\underline{k} = \left(k_1\underline{1}'_{b_1}, k_2\underline{1}'_{b_2}, 2\underline{1}'_{s(s-2)}\right)$

Let us consider the matrix M_0 given by Calinski, 1971

$$M_0 = R^{-1}NK^{-1}N' - \frac{1}{n}1_v r',$$

where n is the number of experimental units and $n = \sum_{i=1}^{v} r_i = \sum_{j=1}^{b} k_j$

For the incidence matrix given in equation (1) the value of M_0 is computed as

$$M_0 = \frac{1}{\left(12s(s-1)^2\right)} \begin{bmatrix} \alpha & -\beta & -\beta & -\beta & \dots & -\beta & -\gamma \\ -\beta & \alpha & -\beta & -\beta & \dots & -\beta & -\gamma \\ -\beta & -\beta & \alpha & -\beta & \dots & -\beta & -\gamma \\ -\beta & -\beta & -\beta & \alpha & \dots & .-\beta & -\gamma \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\beta & -\beta & -\beta & -\beta & \dots & \alpha & -\gamma \\ -\delta & -\delta & -\delta & -\delta & \dots & -\delta & \zeta \end{bmatrix}_{(v\ \mathrm{x}\ v)}$$

where $\alpha = (s-1)(2s^2 - 5s + 9)$; $\beta = (s^2 - 2s + 5)$ $\gamma = (s-1)(s^2 - 3s + 4)$;

$\delta = (3/s)(s-1)(s^2 - 3s + 4)$; $\zeta = 3(s-1)(s^2 - 3s + 4)$

$$M_0 = \frac{1}{12s(s-1)^2} \begin{bmatrix} \left\{(\alpha+\beta)I_{(v-1)} - \beta E_{(vxv)}\right\} & -\gamma \\ & \\ -\delta 1'_{v-1} & \zeta \end{bmatrix}$$

M_0 Matrix has two distinct non-zero eigen values which are

$\theta_1 = \dfrac{s(s^2-3s+6)-2}{6s(s-1)^2}$ with multiplicity $(v-2)$ and

and $\theta_2 = \dfrac{(s^2-3s+4)}{3s(s-1)}$ with multiplicity 1

Hence the incidence matrix given in (1) gives the partially efficiency balanced block design with efficiency factors $E = 1-\mu_1 = \dfrac{s^2(5s-9)+2}{6s(s-1)^2}$ with multiplicity $(v-2)$ and $E_2 = 1-\mu_2 = \dfrac{2(s^2-2)}{3s(s-1)}$ with multiplicity 1. In this design s – blocks are repeated once.

Numerical Illustrations 1

For $s=4$, let us consider two BIBDs D_1 with the parameters $v_1 = b_1 = 4$; $r_1 = k_1 = 3$; $\lambda_1 = 2$ and D_2 with the parameters $v_2 = b_2 = 5$; $r_2 = k_2 = 4$; $\lambda_2 = 3$. Then the following incidence matrix N gives the PEB block design with the parameters

$v=5$,$b=17$, $\underline{r}=(9\underline{1}_4^{'},12)$, $\underline{k}=(3\underline{1}_4^{'},4\underline{1}_5^{'},2\underline{1}_8^{'})$ and $\lambda=\lambda_1+\lambda_2=5$

$$N=\begin{bmatrix} 1&1&1&0&1&1&1&1&0&1&0&0&0&1&0&0&0\\ 1&1&0&1&1&1&1&0&1&0&1&0&0&0&1&0&0\\ 1&0&1&1&1&1&0&1&1&0&0&1&0&0&0&1&0\\ 0&1&1&1&1&0&1&1&1&0&0&0&1&0&0&0&1\\ 0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

The $P=NK^{-1}N'$ matrix is computed as

$$\begin{bmatrix} 3&17/12&17/12&17/12&7/4\\ 17/12&3&17/12&17/12&7/4\\ 17/12&17/12&3&17/12&7/4\\ 17/12&17/12&17/12&3&7/4\\ 7/4&7/4&7/4&7/4&5 \end{bmatrix}$$

The $M_{01}=R^{-1}NK^{-1}N'$ matrix is computed as

$$\begin{bmatrix} 1/3&17/108&17/108&17/108&7/36\\ 17/108&1/3&17/108&17/108&7/36\\ 17/108&17/108&1/3&17/108&7/36\\ 17/108&17/108&17/108&1/3&7/36\\ 7/48&7/48&7/48&7/48&5/12 \end{bmatrix}$$

The computed $M_{02}=\frac{1}{n}1_v r'$ matrix is

$$\begin{bmatrix} 3/16&3/16&3/16&3/16&1/4\\ 3/16&3/16&3/16&3/16&1/4\\ 3/16&3/16&3/16&3/16&1/4\\ 3/16&3/16&3/16&3/16&1/4\\ 3/16&3/16&3/16&3/16&1/4 \end{bmatrix}$$

Finally the computed $M_0=R^{-1}NK^{-1}N^{'}-\frac{1}{n}1_v r^{'}$, matrix is

$$\begin{bmatrix} 7/48&-13/432&-13/432&-13/432&-1/18\\ -13/432&7/48&-13/432&-13/432&-1/18\\ -13/432&-13/432&7/48&-13/432&-1/18\\ -13/432&-13/432&-13/432&7/48&-1/18\\ -1/24&-1/24&-1/24&-1/24&1/6 \end{bmatrix}$$

The eigen values of the above matrix are

$$\theta=\begin{pmatrix}0 & 2/9 & 19/108 & 19/108 & 19/108\end{pmatrix}$$

The non-zero eigen values of the above matrix M_0 of the design D is $\theta_1=\frac{19}{108}=0.1759$ with multiplicity $3(=v-2)$ and $\theta_2=\frac{2}{9}=0.2222$ with multiplicity 1.

So the design is PEB with efficiency factors

1. $E_1=1-\mu_1$ = 1 – 0.1759 = 0.8241 with multiplicity $v-2=3$ and
2. $E_2=1-\mu_2$ = 1 – 0.2222 = 0.7778 with multiplicity 1

In this design s-blocks are repeated once.

Theorem 2

Consider a BIBD design D_1 with the parameters $v_1 = b_1 = s, r_1 = k_1 = s-1,\ \lambda_1 = s-2$ where s is any positive integer (≥ 3). Now consider the incidence matrix N defined as

$$N = \begin{bmatrix} (N_1)_{v_1 \times b_1} & E_{v_1 \times 2} & I_{v_1} & I_{v_1} \\ 1_{1\times b_1} & 0_{1\times 2} & 1_{1\times b_1} & 0_{1\times b_1} \\ 1_{1\times b_1} & 0_{1\times 2} & 0_{1\times b_1} & 1_{1\times b_1} \end{bmatrix} \tag{2}$$

gives the un-replicated partially efficiency balanced design with $(v_1 + 2)$ treatments.

Here N_1 is the incidence matrix of the BIBD design D_1; E is the matrix of order $(v_1 + 2)$ whose all elements are one; $1_{1\times b_1}$ is the unit vector of order $(1 \times b_1)$; $0_{1\times b_1}$ is the null vector.

Then the design with the incidence matrix N gives the unequally replicated PEB block design with repeated blocks having the following parameters:

$$v = (v_1 + 2); b = 3b_1 + 2;\ \underline{r} = \left((r_1 + 4)\underline{1}'_{v_1}, 2b_1\underline{1}'_2\right);\ \underline{k} = \left((k_1 + 2)\underline{1}'_{b_1}, v_1\underline{1}'_2, 2\underline{1}'_{2v_1}\right)$$

Proof

The incidence matrix defined in equation (2) consists of $(v_1 + 2)$ rows and considering these as treatments obviously the numbers of treatments are $v = (v_1 + 2)$. Also it is obvious that, the above incidence matrix consists of $b_1 + 2(v_1 + 1)$ columns and considering these as the blocks the number of blocks are $b = b_1 + 2(v_1 + 1)$ or $b = b_1 + 2(b_1 + 1) = 3b_1 + 2$. So the order of the N matrix is(s+2)x (s+2).

In the above incidence matrix out of $(v_1 + 2)$ rows; in v_1 rows 1 occurs $(r_1 + 4)$ times and in the remaining two rows 1 occurs $2b_1$ times and hence the number of replications is

$$\underline{r} = \left(r_1^* \underline{1}'_{v_1}, r_2^* \underline{1}'_2\right) \ \textit{where}\ r_1^* = r_1 + 4 \text{ and } r_2^* = 2b_1$$

Also in the above incidence matrix, out of $b_1 + 2(v_1 + 1)$ columns, in b_1 columns 1 appears $k_1 + 2$ times; in two columns 1 appears v_1 times and in $2v_1$ columns 1 appears two times. Hence the block size is

$$\underline{k} = \left((k_1 + 2)\underline{1}'_{b_1}, v_1\underline{1}'_2, 2\underline{1}'_{2v_1}\right);\ v = s + 2;\ b = 3s + 2$$

The $P = NK^{-1}N'$ matrix is computed as

$$\begin{bmatrix} \frac{2(s^2+s+1)}{s(s+1)} & \frac{s^2+2}{s(s+1)} & \cdots & \frac{s^2+2}{s(s+1)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \frac{s^2+2}{s(s+1)} & \frac{2(s^2+s+1)}{s(s+1)} & \cdots & \frac{s^2+2}{s(s+1)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{s^2+2}{s(s+1)} & \frac{s^2+2}{s(s+1)} & \cdots & \frac{2(s^2+s+1)}{s(s+1)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \frac{(3s-1)(s+3)}{4s(s+1)} & \frac{(3s-1)(s+3)}{4s(s+1)} & \cdots & \frac{(3s-1)(s+3)}{4s(s+1)} & \frac{s^2+3s}{2(s+1)} & \frac{s}{(s+1)} \\ \frac{(3s-1)(s+3)}{4s(s+1)} & \frac{(3s-1)(s+3)}{4s(s+1)} & \cdots & \frac{(3s-1)(s+3)}{4s(s+1)} & \frac{s}{(s+1)} & \frac{s^2+3s}{2(s+1)} \end{bmatrix}$$

The order of the above matrix is (s+2) x (s+2)

The computed $M_{01} = R^{-1}NK^{-1}N'$ matrix is

$$\begin{bmatrix} \frac{2(s^2+s+1)}{s(s+1)(s+3)} & \frac{s^2+2}{s(s+1)(s+3)} & \cdots & \frac{s^2+2}{s(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \frac{s^2+2}{s(s+1)(s+3)} & \frac{2(s^2+s+1)}{s(s+1)(s+3)} & \cdots & \frac{s^2+2}{s(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{s^2+2}{s(s+1)(s+3)} & \frac{s^2+2}{s(s+1)(s+3)} & \cdots & \frac{2(s^2+s+1)}{s(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} & \frac{s(3s-1)}{(s+1)(s+3)} \\ \frac{(3s-1)}{4s(s+1)} & \frac{(3s-1)}{4s(s+1)} & \cdots & \frac{(3s-1)}{4s(s+1)} & \frac{s^2+3s}{4s(s+1)} & \frac{s}{(s+1)} \\ \frac{(3s-1)}{4s(s+1)} & \frac{(3s-1)}{4s(s+1)} & \cdots & \frac{(3s-1)}{4s(s+1)} & \frac{s}{(s+1)} & \frac{s^2+3s}{4s(s+1)} \end{bmatrix}$$

The order of the above matrix is (s+2) x (s+2)

The computed $M_{02} = \frac{1}{2} I_v r'$ matrix is

$$\frac{1}{s^2+7s}\begin{bmatrix} s+3 & s+3 & \cdots & s+3 & 2s & 2s \\ s+3 & s+3 & \cdots & s+3 & 2s & 2s \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ s+3 & s+3 & \cdots & s+3 & 2s & 2s \end{bmatrix}$$

Finally the computed $M_0 = R^{-1}NK^{-1}N' - \frac{1}{n}1_v r'$, matrix is

$$\begin{bmatrix} \alpha & -\beta & \cdots & -\beta & -\gamma & -\gamma \\ -\beta & \alpha & \cdots & -\beta & -\gamma & -\gamma \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -\beta & -\beta & \cdots & \alpha & -\gamma & -\gamma \\ -\delta & -\delta & \cdots & -\delta & \alpha_1 & -\beta_1 \\ -\delta & -\delta & \cdots & -\delta & -\beta_1 & \alpha_1 \end{bmatrix}$$

where

$$\alpha = \frac{s^3+9s^2+s+5}{(s+1)(s+3)(s^2+7s)},\ \beta = \frac{13s-5}{(s+1)(s+3)(s^2+7s)},\ \gamma = \frac{(s^2-4s+19)}{2(s+1)(s+3)(s+7)}$$

$$\delta = \frac{(s^2-4s+19)}{4(s+1)(s^2+7s)},\quad \alpha_1 = \frac{s^2+2s+13}{4(s+1)(s+7)},\quad \beta_1 = \frac{3s(s-1)}{2(s+1)(s^2+7s)}$$

The non-zero eigen value of the above matrix is as follows

$$\theta = [0 \quad \underbrace{\theta_1 \quad \cdots \quad \theta_1}_{(v-s)\ \ times} \quad \theta_2 \quad \theta_3]$$

where

$$\theta_1 = \alpha + \beta = \frac{(s^3+9s^2+14s)}{(s+1)(s+3)s(s+7)}$$ with multiplicity (v-s)

$$\theta_2 = \alpha_1 - \beta_1 + 2\gamma = \frac{\left(s^3 + 3s^2 - 9s + 133\right)}{4(s+1)(s+3)(s+7)} \text{ with multiplicity 1}$$

$$\theta_3 = \alpha_1 + \beta_1 = \frac{\left(s^2 + 8s + 7\right)}{4(s+1)(s+7)} \text{ with multiplicity 1}$$

Hence the incidence matrix given in (2) gives the partially efficiency balanced design with efficiency factors $E_1 = 1 - \mu_1 = \frac{s^3 + 10s^2 + 22s + 7}{(s+1)(s+3)(s+7)}$ with multiplicity $\left(v-2\right)$; $E_2 = 1 - \mu_2 = \frac{3s^3 + 14s^2 + 133s - 49}{4(s+1)(s+3)(s+7)}$ with multiplicity 1 and $E_3 = 1 - \mu_3 = \frac{3s^2 + 24s + 21}{4(s+1)(s+7)}$ with multiplicity 1.

Numerical illustration 2

For s=4, let us consider a BIBD with $v_1 = b_1 = s$ =4, $r_1 = k_1 = s-1$ =3 and $\lambda_1 = s-2$ =4 – 2 = 2, then the incidence matrix N as per the theorem 2 gives PEB block design with the following parameters v=s+2 = 4+2 = 6; b=3s+2 = 14; $\underline{r} = (7\underline{1}_4^{'}, 8\underline{1}_2^{'})$ and $\underline{k} = (5\underline{1}_4^{'}, 4\underline{1}_2^{'}, 2\underline{1}_8^{'})$.

$$N = \left[\begin{array}{cccc|cc|cccc|cccc}
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\hline
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{array}\right]_{6\times 14}$$

Here obviously v=s+2 = 4+2 = 6, b=14, $\underline{r} = (7\underline{1}_4^{'}, 8\underline{1}_2^{'})$ and $\underline{k} = (5\underline{1}_4^{'}, 4\underline{1}_2^{'}, 2\underline{1}_8^{'})$.

The $P = NK^{-1}N'$ matrix is computed as

$$\left[\begin{array}{cccc|cc}
\frac{21}{10} & \frac{9}{10} & \frac{9}{10} & \frac{9}{10} & \frac{11}{10} & \frac{11}{10} \\
\frac{9}{10} & \frac{21}{10} & \frac{9}{10} & \frac{9}{10} & \frac{11}{10} & \frac{11}{10} \\
\frac{9}{10} & \frac{9}{10} & \frac{21}{10} & \frac{9}{10} & \frac{11}{10} & \frac{11}{10} \\
\frac{9}{10} & \frac{9}{10} & \frac{9}{10} & \frac{21}{10} & \frac{11}{10} & \frac{11}{10} \\
\hline
\frac{11}{10} & \frac{11}{10} & \frac{11}{10} & \frac{11}{10} & \frac{14}{5} & \frac{4}{5} \\
\frac{11}{10} & \frac{11}{10} & \frac{11}{10} & \frac{11}{10} & \frac{4}{5} & \frac{14}{5}
\end{array}\right]_{6\times 6}$$

The $M_{01} = R^{-1}NK^{-1}N'$ matrix is computed as

$$\left[\begin{array}{cccc:cc}
\frac{3}{10} & \frac{9}{70} & \frac{9}{70} & \frac{9}{70} & -\frac{19}{770} & -\frac{19}{770} \\
\frac{9}{70} & \frac{3}{10} & \frac{9}{70} & \frac{9}{70} & -\frac{19}{770} & -\frac{19}{770} \\
\frac{9}{70} & \frac{9}{70} & \frac{3}{10} & \frac{9}{70} & -\frac{19}{770} & -\frac{19}{770} \\
\frac{9}{70} & \frac{9}{70} & \frac{9}{70} & \frac{3}{10} & -\frac{19}{770} & -\frac{19}{770} \\
\hdashline
\frac{11}{80} & \frac{11}{80} & \frac{11}{80} & \frac{11}{80} & \frac{7}{20} & \frac{1}{10} \\
\frac{11}{80} & \frac{11}{80} & \frac{11}{80} & \frac{11}{80} & \frac{1}{10} & \frac{7}{20}
\end{array}\right]_{6\times 6}$$

The $M_{02} = \frac{1}{n} 1_v r'$ matrix is computed as

$$\begin{bmatrix}
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11} \\
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11} \\
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11} \\
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11} \\
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11} \\
\frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{7}{44} & \frac{2}{11} & \frac{2}{11}
\end{bmatrix}_{(6x6)}$$

Finally the computed $M_0 = R^{-1} N K^{-1} N' - \frac{1}{n} 1_v r'$, matrix is

$$\left[\begin{array}{cccc:cc}
\frac{31}{220} & -\frac{17}{557} & -\frac{17}{557} & -\frac{17}{557} & -\frac{19}{770} & -\frac{19}{770} \\
-\frac{17}{557} & \frac{31}{220} & -\frac{17}{557} & -\frac{17}{557} & -\frac{19}{770} & -\frac{19}{770} \\
-\frac{17}{557} & -\frac{17}{557} & \frac{31}{220} & -\frac{17}{557} & -\frac{19}{770} & -\frac{19}{770} \\
-\frac{17}{557} & -\frac{17}{557} & -\frac{17}{557} & \frac{31}{220} & -\frac{19}{770} & -\frac{19}{770} \\
\hdashline
-\frac{19}{880} & -\frac{19}{880} & -\frac{19}{880} & -\frac{19}{880} & \frac{37}{220} & -\frac{9}{110} \\
-\frac{19}{880} & -\frac{19}{880} & -\frac{19}{880} & -\frac{19}{880} & -\frac{9}{110} & \frac{37}{220}
\end{array}\right]_{6\times 6}$$

The eigen values of the above M_0 matrix has three distinct non-zero eigen values

$$\theta = \begin{bmatrix} 0 & \frac{60}{35} & \frac{60}{35} & \frac{60}{35} & \frac{19}{140} & \frac{1}{4} \end{bmatrix}$$

The non-zero eigen values of the above matrix M_0 of the design D is $\theta_1 = \frac{60}{35}$ with multiplicity 3 (= v-s+1); $\theta_2 = \frac{19}{140}$ with multiplicity 1 and $\theta_3 = \frac{1}{4}$ with multiplicity 1.So the design is PEB with efficiency factors

1. $E_1 = 1 - \mu_1$ = 1 – 0.1714 = 0.8286 with multiplicities 3

2. $E_2 = 1 - \mu_2$ = 1 – 0.1357 = 0.8643 with multiplicity 1
3. $E_3 = 1 - \mu_3$ = 1 – 0.2500 = 0.7500 with multiplicity 1

Applications

Balanced incomplete block designs and partially incomplete block designs are not available for all the parameters because of its parametric relations. Similarly, variance balanced designs are not possible for the required parameters. In such situation, efficiency balanced designs are very much useful. In agricultural plant breeding trails as the numbers of treatments are increased, it is not possible to maintain the homogeneity within the block and some treatment groups effects are estimated with different efficiency factor, in such a situations partially efficiency designs are very much useful and desirable one.

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