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Research Article

REVIEW ON BEHAVIOUR OF UNDER-REINFORCED SHALLOW FIBROUS CONCRETE SOLID BEAMS UNDER PURE TORSION

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ABSTRACT

In this paper, the methods for evaluating torsional strength of fibrous concrete beams are reviewed and highlighted the basic assumptions in these methods.The variables which include in the proposed model in these methods for predicting of torsional strength in shallow under-reinforced and plain fibrous concrete beams are explained. The proposed models indicated that volume fraction of steel fibre, aspect ratio of fibres, tensile strength of fibres, geometry of the section and inclination angle of spiral crack have an impact on the torsional strength in pre-cracking stage, whereas longitudinal and transverse reinforcement ratios, enclosed area by stirrups as well as stirrup spacing have influence on the ultimate torsional strength.Likewise, the impact of these variables on the torsional strength in prior of cracking and post-cracking stages improved by the experimental researcher's .It was noticed that there is a gap between assumption and experimental variables. While in space truss analogy method, the enclosed area by stirrups is considered in the prediction of ultimate torsional strength, whereas according to the basic assumptions the same area is ignored. Likewise, the area of concrete outside of stirrups is considered as a basic assumption whereas this area is completely ignored in the evaluating ultimate torsional strength. Moreover, the material's strength is limited in the predicting models. Consequently, these limits make an efficient torsion design of fibrous concrete beams difficult.

INTRODUCTION

The research works on theshallow plain and under-reinforced fibrous concrete beams with different grades subjected to pure torsion had been carried out since 1976. It is realized that there is shortcoming between variables, which are considered in the different theories, and the experimental variables. There are many works reported on shallow fibrous concrete beams under pure torsion, which are divided into two parts. The first part of research deals with the plain fibrous concrete beams, including different grade of concrete such as steel fibre normal strength concrete and steel fibre high strength concrete. The shape and depth-to-width ratio of the section, the properties of steel fibres and compressive strength of concrete were the main experimental variables, which confirmed that they had an impact on the torsional capacity of the beams. The second part of research deals with the under-reinforced fibrous concrete beams, which had the different grades such as steel fibre normal strength concrete and ultra high performance fibre reinforced concrete. The type of both types of reinforcement by steel fibres were the important experimental variables. In brief, the variables used in the previous works can be observed in Table 1-2.

Table 1 Summary of experimental works on plain fibrous normal and high strength shallow concrete beams under pure torsion

References	**Experimental variables**					
	V_f	**L_f**	**λ**	**$f_{c'}$**	**N3**	**N6**
Plain fibrous normal strength shallow concrete beams under pure torsion						
Hafeez Khan *et al*. (1976)	▪		▪			
Mansur and Paramansivam (1982)	▪	▪				
Tegas (1989)	▪		▪	▪		
Wafa *et al*. (1992)	▪					
Karayannis (2000)	▪		▪	▪		▪
Chalioris and Karayannis (2009)	▪					▪
Plain fibrous normal and high strength shallow concrete beams under pure torsion						
Narayanan and Toorani-Goloosalar (1979)				▪		
Narayanan and Green (1980)	▪		▪	▪		
Narayanan and Kareem-Palanjian (1983)	▪		▪		▪	▪
Craig *et al*.(1986)	▪	▪				
GunneswaraRao and Seshu (2003)	▪			▪		

N3: depth to width ratio N6: shape of section

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Table 2 Summary of experimental works on under-reinforced fibrous normal strength shallow concrete beams under pure torsion

References	ρ_L	ρ_t	R_L	R_t	V_f	λ	N1	N2	N3	N4	N5
Under-reinforced fibrous normal strength shallow concrete beams under pure torsion											
Craig *et al.* (1984)					▪	▪	▪				
Narayanan and Kareem-Palanjian (1986)	▪	▪	▪	▪	▪				▪		
Mansur *et al.* (1989)	▪	▪			▪						
El-Niema (1993)	▪	▪			▪						
Namiq (2012)					▪			▪	▪	▪	
Fuad Okay and SerkanEngin (2012)	▪				▪	▪					
Raut and Kulkarni (2014)					▪						
Under-reinforced ultra-high performance fibre reinforced shallow concrete solid beams under pure torsion											
Fehling and Ismail (2012)	▪	▪			▪						▪
Joh *et al.* (2012)	▪	▪									
Empelmman and Oettel (2012)	▪	▪			▪						
Yang *et al.* (2013)	▪	▪			▪						
Ismail, M. (2015)			▪	▪	▪						▪

N1: stirrup spacing N2: section types (hollow/ solid)
N4: different shape of section N5: different type of fibre

There are many theories to analyze the behaviour of fibrous concrete beams under pure torsion, which are divided into two stages (prior of cracking and post-cracking) for plain and under reinforced fibrous concrete beams respectively. For analysis of plain fibrous concrete beams elastic theory, plastic theory, skew bending theory, empirical modelsand space truss analogywere used. The main theoretical variables cross section dimensions, volume fraction of steel fibre, aspect ratio of fibres, length of fibres, diameter of fibres, bond factor, fibre orientation, ratio of average fibre stress to maximum fibre stress, compressive strength of fibrous concrete, modulus of rupture of fibrous concrete, splitting tensile strength of fibrous concrete and shear stress of fibrous concretewere considered. In contrast, in the under-reinforced fibrous concrete beams analysis elastic theory, skew bending theory and space truss analogywere used. The main theoretical variables were adopted to have an impact on the torsional strength which were the dimension of section, thickness of shear flow zone, area enclosed by centre line of shear flow zone, volume fraction of steel fibres, length, diameter, bond factor of steel fibres, compressive strength, splitting tensile strength, modulus of rupture of fibrous concrete, longitudinal and transverse reinforcement ratios and their yield strengths, stirrup spacing, area enclosed by stirrups, angle of crack inclination and twisting angle at failure. The summary of these variables in each theory are explained in Table 3.

In general, in spite of the fact that there are some variables having impact on the torsional strength, which confirmed practically, these variables are ignored in proposed models for predicting torsional capacity. In the same way, there are some variables that have influence on the torsional capacity, which verified theoretically, there are not considered as an experimental variables.

Research significance

This paper highlights the variables, which affected the torsional capacity at post-cracking stage and the other variables that affected torsional strength at pre-cracking stage in the fibrous concrete beams subjected to pure torsion.

Table 3 Variables were taken in the different theories versus experimental researches for shallow fibrous concrete beams under pure torsion in pre-cracking and post-cracking stages

Zone of research	Variables	Pre-cracking stage						Post-cracking stage					
		ET	PT	SBT	STA	EASEM	ER	ET	PT	SBT	STA	EASEM	ER
Section of beams	Shape of the section						√						√
	Dimension of the cross section	√	√	√		√	√	√		√	√		√
	Thickness of shear flow zone										√		
	Area enclosed by centre line of shear flow zone										√		
Properties of steel fibre	Volume fraction of steel fibre	√		√			√	√			√		√
	Aspect ratio of steel fibre	√		√			√						√
	Length of fibre	√		√			√				√		
	Diameter of fibre	√		√							√		
	Bond factor of steel fibre	√		√							√		
	Different types of steel fibre						√						√
	Fibre orientation	√											
	Average fibre stress to maximum fibre stress ratio	√											
Properties of fibrous concrete	Compressive strength			√		√	√			√	√		
	Split tensile strength			√		√					√		
	Modulus of rupture	√	√	√							√		
	Shear stress	√						√					
Longitudinal reinforcement properties	Longitudinal reinforcement ratio							√			√		√
	Yield strength										√		
	Replacement of longitudinal reinforcement by steel fibre												√
Transverse reinforcement properties	Transverse reinforcement ratio							√			√		√
	Yield strength										√		
	Replacement of transverse reinforcement by steel fibre									√	√		√
	Stirrup spacing									√	√		
	Area enclosed by stirrups									√	√		
	Area of transverse reinforcement												√
Crack pattern	Crack inclination angle							√			√		
Failure pattern	Twisting angle at failure							√					

Even though there are studies related to fibrous concrete beams under pure torsion, the gaps are still there between basic assumption and outcome models from the different theories. The recommendations are given as well for improving the prediction models for evaluating the torsional capacity at post-cracking and pre-cracking stages.

LITERATURE REVIEWS

Shallow plain fibrous concrete beam under pure torsion

Plain shallow fibrous normal strength concrete beam under pure torsion

It is believed that the compressive strength of fibrous concrete has an impact on the torsional capacity (Karatanan 2000 and Tegas1989); experimentally it is not found any evidence about this impact due to the tensile strength of fibrous concrete responsible for resisting torsion. It was confirmed by Tega (1980) that the maximum size of aggregate did not influence the torsional strength due to unclear relation between tensile strength and the size of aggregate. Moreover, even though the torsional strength of fibrous concrete beams is improved by inclusion of steel fibre (Chalioris 2009; Mansur and Paramasivan 1982; Karayanan 2000; Tegas 1989 and Hafeez *et al*. 1976), it was concluded that there was not significant changes in the torsional strength when the volume fraction of fibres less than 1% (Wafa *et al*. 1992). In addition to volume fraction of fibre, it was pointed out that the relationship between aspect ratio and torsional capacity was obsereved to be nearly linear (Hafeez *et al*. 1976; Tegas1980; Mansur and Paramasivan 1982).

The latter confirmed that while the aspect ratio of fibres remains constant, the torsional resistance increases with increase in fibre length. The most important point was the shape of the section which improved the torsional capacity for flanged shape more than rectangular one due to fibre orientation effect (Chalioris and Karayannis2009).

The best proposed models for evaluating torsional resistance are based on different theories such as elastic, plastic and skew bending. The reliable proposed models were (7, 8, 9, 10), which state in Appendix B, based on elastic theory Tegas (1989) while a good model (6), which states in Appendix B, based on skew bending theory was proposed by Wafa *et al*. (1992). Likewise, the proposed model (12), which stated in Appendix B, basing on plastic theory was a good for evaluating torsional strength (GunneswaraRao and Seshu2003).

Plain shallow fibrous normal and high strength concrete beams under pure torsion

Experimentally, there are no direct relation between torsional capacity of the beam and compressive strength of fibrous concrete, even though it is considered as an experimental variable in some researches (Narayanan and Green 1980; Narayanan and Toorani-Goloosalar1979).

Due to inclusion of steel fibres, the tensile strength of fibrous concrete is improved significantly. Therefore, there is a proportional relation between volume fraction of steel fibres and torsional capacity via tensile strength (Craig *et al*. 1986 and Narayanan and Kareem-Palanjian1983). In addition to the improvement of torsional strength, the ductility, stiffness and amount of cracks increased due to inclusion of steel fibres (GunneswaraRao and Seshu 2003 and Craig *et al* 1986).

There are several methods for evaluating torsional capacity in normal and high strength fibrous concrete beams subjected to pure torsion. The more applicable models were (19, 20, 21), which state in Appendix B, based on skew bending theory (Narayanan and Kareem-Palanjian1983). Furthermore, it was nominated that the more reliable semi-empirical formula was model (16), which states in Appendix B, for predicting torsional resistance (GunneswaraRao and Seshu2003).

Under- reinforced shallow fibrous concrete beams under pure torsion

Under- reinforced fibrous normal strength concrete shallow beams under pure torsion

The torsional capacity of under-reinforced fibrous concrete beamsis found to be strongly dependent on the amount and type of fibres. Meanwhile, amount of fibre was improved each of stiffness, ultimate torsional capacity, ductility, rotational capacity and increased the number of cracks then reduced the crack width (Raut and Kulkarni 2014; Namiq 2012; Narayanan and Kareem-Palanjian 1986 and Craig *et al*. 1984). In addition, the cracking strength and torsional fracture energy improved by inclusion of steel fibres which confirmed that the energy absorption capacity is significantly affected by the addition of steel fibres ((El-Niema1993; Fuad Okay and SerkanEngin 2012).

Even though the inclusion of steel fibres has these positive influences on the capacity of beam for resisting torque, increasing the amount of longitudinal reinforcement with a fixed volume fraction of fibres for beams without stirrups had negligible influence on the torsional strength, ductility and toughness. Likewise, fibre reinforced concrete beams with or without continuous reinforcement failed eventually by bending about a skew axis (Mansur *et al*. 1989).

In spite of the fact that the longitudinal and transverse reinforcements were the important elements for improving torsional strength by adding steel fibres (Mansur *et al*. 1989), the partial or full replacement of transverse reinforcement by steel fibre improved the ultimate torsional capacity, while the replacement of longitudinal reinforcement by steel fibre had smaller impact on the torsional strength (Narayanan and Kareem-Palanjian1986).

There are many theories use for evaluating torsional capacity of under-reinforcement fibrous concrete beams. The useful models were (30, 31, 32, and 33), which issue in Appendix B, proposed based on space truss analogy (Narayanan and Kareem-Palanjian1986). In addition to this model, there was a valid model (34), which states in Appendix B, depending on skew bending theory for evaluating torsional resistance as an alternative (Craig *et al*. 1984). Likewise, there was an empirical model (25) and (24), which state in Appendix B. The latter model based on elastic theory was proposed to predict the torsional capacity in pre-cracking and post-cracking stages respectively and it had variance 16% from the experimental result (Fuad Okay and SerkanEngin 2012).

Under-reinforced ultra high performance fibre reinforced concrete shallow beams under pure torsion

Empelmann and Oettel (2012) concluded that the crack width is reduced and the numbers of cracks are increased due to adding steel fibres. Consequently, the cracking torque, ultimate torsional strength and torsional stiffness after cracking were increased (Ismail 2015; Yang *et al*. 2013). The other variable which has impact on the torsional strength was transverse reinforcement ratio, which causes an increase in ultimate torsional strength as well as increasing in longitudinal reinforcement (Yang *et al*. 2013). Moreover, the addition of the longitudinal reinforcement to the steel fibre ultra high performance concrete beams with low percentage of volume fraction of steel fibre, the stiffness incredibly improved after cracking and increased the average number of crack per meter of length and reduced the crack width as well (Fehling and Ismail 2012). In addition, the UHPFRC beams with longitudinal reinforcement only showed the ductile behavior, whereas the specimens with both of the longitudinal and transverse reinforcement showed hardening after cracking (Ismail 2015; Joh *et al*. 2012).

Joh *et al.* (2012) proposed models (35, 36, 37 and 38), which state in Appendix B, based on the space truss analogy to predict the torsional strength in pre-cracking and post-cracking stage. They concluded that the modified models seem to work reasonably. In addition, it was proposed a mathematical equation from the space truss models to determine the loading capacity of the tension strut of the fibre, which showed a good correlation with regard to the experimental result (Empelmann and Oettel 2012).

APPLICATION AND DISCUSSION

Experimental works

This section highlights the previous experimental works, which have been carried out on the factors that affect fibrous concrete beams under pure torsion.

Plain fibrous concrete shallow beams subjected to pure torsion

The main experimental variables affect the cracking torsional strength in the different researches focus on properties of steel fibres, properties of materials inside the concrete, geometry of the section such as the cross-sectional shape and aspect ratio of the section and mechanical properties of fibrous concrete, which include compressive strength, split tensile strength, and modulus of rupture. There are many variables which influence on short mechanical properties such as volume fraction of steel fibres, aspect ratio of steel fibresand length of steel fibres. The impact of each variable on the torsional strength could be explained as follows:

Volume fraction of steel fibres

In general, the torsional strength increases with an increase in volume fraction of steel fibre but in different extent based on their volume percentage in the concrete. The fibre-reinforced concrete is classified into three types depending on volume fraction of steel fibres which are low, moderate, and high volume fraction of steel fibre. The effect of the ratio of steel fibres on torsional resistance can be summarized as follows:

Fibre reinforced concrete with low steel fibre volume fraction

The torsional strength of fibre reinforced concrete with low steel fibre volume fraction is improved from inclusion of steel fibre with low steel fibre volume fraction. For instance, it is confirmed that increasing in fibre content upto 1% caused an increase in the torsional strength up to 29% (Wafa *et al*. 1992).

Fibre reinforced concrete with high steel fibre volume fraction

The torsional strength improved up to 27% compared to beams without steel fibres with respect to add of steel fibre of 3% (Mansur and Paramasivan1982). Moreover, the value of improvement in the range of 34.4% was confirmed by (Chaliories and Karayannis2009). In addition, the amount of improvement reached 33.93% (Karayannis2000). In spite of the fact that adding steel fibre causes an increase in torsional strength, the combined action of inclusion of steel fibres with increasing their aspect ratio caused an extra increase in torsional strength up to 80% (Tegas1989).

Aspect ratio of steel fibres

The torsional strength of beams improved up to 37% due to an increase in aspect ratio of steel fibres from 40 to 75, while the volume fraction of steel fibre kept constant at 2% (Tegas1989). In fact, this value of increasing in torsional strength has verified that the torsional strength increased up to 20% own to increasing the aspect ratio of steel fibre from 27 to 78 while volume fraction kept constant at 3% (Mansur and Paramasivan1982). It could be concluded that the influence of the aspect ratio on the torsional strength decreases with increase in volume fraction of steel fibres.

Maximum aggregate size

There is no confirmation from the researchers about the relation between maximum sizes of aggregate or torsional strength at pre-cracking stage. This was confirmed by Tegas (1989) that the maximum size of aggregate had not affected on the torsional strength in the elastic region.

Compressive strength of fibrous concrete

In general, there is a proportional relation between torsional capacity and compressive strength of fibrous concrete, which relates directly with tensile strength. This relation confirmed that the torsional strength of beams improved up to 40.80% due to increasing in compressive strength up to 126.7%, keeping the volume fraction of steel fibre constant at 0.9%, which has been the maximum dosage to give this amount of improvement (Tegas1989).

Geometry of the section

The torsional strength of flanged beams seemed to be increased with respect to the rectangular ones by 45%, keeping the percentage of steel fibres constant. That is due to the impact of inclination of steel fibres (Chalioris and Karayannis2009).

Under-reinforced fibrous concrete shallow beams subjected to pure torsion

The ultimate torsional capacity of under-reinforced fibrous concrete beams affects some variables, which keeping include steel fibre properties, transverse and longitudinal reinforcement ratios and the replacement of reinforcement by steel fibres.

Properties of steel-fibres

Volume fraction of steel fibres

The ultimate torsional capacity increased up to 68.75% due to increasein steel-fibre content by around 0.9% keeping the longitudinal and transverse reinforcement ratios constant

(Narayanan and Kareem-Palanjian1986). It was improved considerably due to the addition of steel fibres. An increase in volume fraction of steel fibres in the range of 1.2% increased the torsional capacity at failure about 66% keeping the both types of reinforcing bars constant. This amount of improvement reduced to half while adding steel fibre 0.6% by volume (El-Niema1993). In addition, the amount of improvement in torsional capacity increased by about 23.55% for inclusion of 1.5% fraction of steel fibres when the both kinds of reinforcement bars were kept constant (Mansur *et al.* 1989). Furthermore, itwas confirmed that torsional capacity improved around 19.02% by adding 0.6% of steel fibre while it was found that ultimate torsional strength was not affected by adding steel fibres in the range of 0.3% by volume (Fuad Okay and SerkanEngin2012). Moreover, it was verified the torsional capacity in UHPFRC beams under pure torsion improved by about 76.3% due to addition of 1.25% of steel fibreskeepingthe amount of reinforcing re-bars constant (Empelmann and Oettel2012). Last but not the least, it was verified that the increasing of volume fraction of steel fibres from 1 to 2 % in UHPFRC caused an increase in torsional strength at failure up to 26.6 % (Yang *et al.* 2013).

Aspect ratio of steel fibres

The ultimate torsional capacity of beams increased up to 5.89% due to increasing in the aspect ratio of steel fibres from 40 to 80, keeping the percentages of steel fibres as well as both types of reinforcing bars constant (Fuad Okay and SerkanEngin2012).

Amount of reinforcement bars

B-2.1 Longitudinal reinforcement ratio:
The ultimate torsional strength increased up to 25.68% due to an increase in longitudinal reinforcement ratio from 0.25% to 0.77%, keepingthe amount of transverse reinforcement constant even though the amount of steel fibres has reduced from 1.11% to 0.59% (Narayanan and Kareem-Palanjian1986). This improvement in torsional resistance confirmed that to be more than that foregoing. For instance, The torsional strength improved up to 40.72% due to an increase in longitudinal reinforcement ratio from 0.63% to 1.26%, keeping volume fraction of steel fibre, compressive strength of the concrete and longitudinal–to-transverse reinforcement ratio constant (Mansur *et al.* 1989).

Further, the ultimate torsional capacity increased up to 31.22% due to increasing in the longitudinal reinforcement ratio from 0.67% to 1.51%, keeping the volume fraction and aspect ratio of steel fibres as well as compressive strength of concrete constant (Fuad Okay and SerkanEngin2012). Moreover, the addition of longitudinal re-bars was effective in improving the post-cracking behaviour of UHPFRC beams, for instance the torsional resistance at failure improved up to 7.45% owing to increasing the longitudinal reinforcement ratio by 0.88% and keeping the other variables constant (Joh *et al.* 2012). Last but not the least; the amount improvement reached to 28% due to an increase in longitudinal reinforcement ratio from 0.56% to 1.27% in UHPFRC beams, keeping the other variables have kept constant (Yang *et al.* 2013).

Transverse reinforcement ratio

In general, there is a proportional relation between ultimate torsional strength and transverse reinforcement ratio. The torsional capacity of beams increased up to 63.45% due to an increase in the amount of transverse reinforcement by 0.35%, keeping the volume fractions of steel fibres and longitudinal reinforcement constant (El-Niema1993). In addition, the torsional strength of UHPFRC beams improved by 20.6% due to increase in the transverse reinforcement ratio by 0.35% (Empelmann and Oettel2012). Further, the torsional capacity increased by 66.11% due to an increase in transverse reinforcement ratio by 0.70%, keeping the other variables constant (Yang *et al.* 2013).

Replacement of reinforcing bars by steel fibres

The torsional capacity slightly increased due to the partial replacement of longitudinal reinforcing bars for steel fibres, whereas the increasing in the ultimate torsional capacity of beams varied between 20 and 35 percentages due to partial or full replacement of stirrups by an equivalent volume fraction of fibres (Narayanan and Kareem-palanjian1986).

Stirrup spacing

The decrease in stirrup spacing from 178 mm to 89 mm increased in the ultimate torsional strength in the range of 27.4% (Craig *et al.* 1984).

Theoretical Models

In this part, theoretical models published are discussed as follows:

Elastic theory

In pre-cracking stage, elastic theory covered each experimental variable from different researches. Therefore, it is the best method for predicting torsional capacity in this stage, whereas it has a lower significance in post-cracking stage because of this theory did not adopt the impact of section shape, different type of steel fibre and replacement of reinforcement by steel fibres.

Plastic theory

The plastic theory for analyzing fibrous concrete beams under pure torsion was a weak theory due to ignoring many variables, which confirmed that they have impact on the torsional strength. For instance, shape of section, properties of steel fibre, which includes volume fraction, aspect ratio, lengthand different types of steel fibres.

Skew bending theory

The skew bending theory was used to analyze fibrous concrete beams under pure torsion which covered the most variables that had an impact on torsional capacity expect shape of section and different types of steel fibres. In spite of the fact that this theory was suitable for predicting torsional capacity in pre-cracking stage, it did not conceal many variables such as transverse and longitudinal reinforcement ratio, shape of section and replacement of reinforcement by steel fibres.

Space truss analogy

The only suitable method for predicting torsional capacity of fibrous concrete beams in pre-cracking stage was the space truss analogy because it covers the majority of variables, which were experimentally verified for their impact on the torsional resistance. Meanwhile, it was ignored influence of different types of steel fibres and the replacement of steel reinforcement by steel fibres.

Empirical and semi-empirical models

These models were slightly weak for predicting torsional capacity of fibrous concrete shallow beams even in pre-cracking stage, because they were only considered the variables like cross sectional dimension and splitting tensile strength which are not sufficient to predict accurate value of torsional resistance.

CONCLUSIONS

The cracking and ultimate torsional strength for different grade of concrete varies according to the variables, which include volume fraction, aspect ratio, length and types of steel fibre, tensile strength of fibrous concrete, shape of cross section, longitudinal and transverse reinforcement ratios, replacement of stirrups as well as longitudinal reinforcement by steel fibres and spacing between stirrups. There are shortcomings in theories to cover every important variable and results presented in previous researches are summarized in the following main points.

1. Although the influence of volume fraction and aspect ratio of steel fibres on the torsional strength in pre-cracking and post-cracking stages was practically verified that they have improved torsional capacity up to 47% and 25% in pre-cracking and post-cracking stages respectively.These variables were ignored in some extent in plastic theory and space truss analogy in both stages.
2. All of the theories for predicting the torsional resistance of fibrous concrete beams are not considered the impact of section shape on the torsional strength, even though the torsional capacity in fibrous concrete flange section was confirmed to be improved 45% more than that in fibrous concrete rectangular section with the same section area.
3. The contribution of longitudinal and transverse reinforcement for improving torsional strength in post-cracking stage is approximately 26% and 66%, respectively. Even though longitudinal reinforcement has significant impact on the torsional resistance, it does not appear in any model for predicting torsional capacity of fibrous concrete beams.
4. The partial replacement of transverse reinforcement by steel fibres improves the torsional strength up to 35% whereas the partial replacement of longitudinal reinforcement make slight improvement in torsional resistance due to large contribution of transverse reinforcement in compare with longitudinal one.
5. The influence of stirrup spacing on torsional capacity in post-cracking stage is ignored by elastic and plastic theories, while it has a role to improve torsional resistance up to 27%.
6. Most of previous works confirmed that the maximum size of aggregate has no effect on the torsional capacity in the pre-cracking stage, because there is unclear relation between maximum size of aggregate and the tensile strength of concrete which has influenced on torsional strength.
7. In spite of the fact that there is proportional relation between the tensile strength of fibrous concrete and torsional strength at pre-cracking stage, the tensile strength has no impact in the post-cracking stage. Further, the tensile strength of concrete related to compressive strength in prediction of torsional strength of concrete beams, and this property involves directly in the diagonal compression strut, which plays a role for main element carrying compression force in the assumed thin-walled tube analogy, but it has ignored in the prediction equation of the same theory.
8. Despite the fact that the predicting model in space truss analogy for evaluating torsional resistance has included the covered area by stirrups, this area is ignored by the same theory as a basic assumption.
9. Even though the centreline of shear flow zone lied on centreline of stirrups as a basic assumption in space truss analogy, half of shear flow zone area was ignored as a minimum for evaluating ultimate torsional capacity.
10. The torsional capacity in space truss analogy model has related significantly with the stress in steel reinforcement even though the stress value in the reinforcement embedded in concrete which was verifiedexperimentally that it has not exceed the half of specified yield strength. Therefore, using the specified yield stressof reinforcement confirmed that the predicting model by space truss model is empirical.

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Appendix A: Notation

Symbol	Description
A_t	One leg area of stirrup, mm^2
A_{oh}	Area enclosed by centre lines of stirrup, mm^2
A_o	$=0.85*A_{oh}$
C_1	Coefficient depends on the aspect ratio of the cross-section of the beam which vary between 0.208 and 0.333
d	Diameter of fibres, mm
d_f	Bond factor
ET	Elastic theory
$EASEM$	Empirical and semi-empirical methods
ER	Experimental researches
F	Fibre factor , $F=\frac{l}{d}*V_f*d_f$
$f_{cf'}$	Compressive strength of steel fibre concrete, N/mm^2
f_r	Modulus of rupture of concrete, N/mm^2
f_{ct}	Splitting tensile strength of concrete, N/mm^2
f_{cr}	Cracking strength of UHPFRC, N/mm^2
f_{rf}	Modulus of rupture of fibrous concrete, N/mm^2
f_{ctf}	Split tensile strength of fibrous concrete, N/mm^2 and it is

	equal to{ $0.51*\sqrt{fc'}+1.80*V_f$ } in model (6) $f_{ctf} = \begin{cases} = 0.65 * f_{mt} \\ = \dfrac{f_{cu}}{20-\sqrt{\lambda * V_f * d_f}} + 0.7 + \sqrt{\lambda *} \end{cases}$ in model (21)
$f_{c'}$	Cube compressive strength of plain concrete, N/mm^2
f_{mt}	Modulus of rupture of plain concrete, N/mm^2
f_t^*	Effective tensile strength of concrete in pure torsion, N/mm^2 $\frac{1}{f_t^*} = \frac{1}{f_{c'}} + \frac{1}{f_{spt}}$
f_t	Tensile strength of fibrous concrete, N/mm^2 $f_t = f_{mt} * V_m + 0.34 * \frac{l}{d} * V_f$
f_{ty}	Yield strength of transverse reinforcement, N/mm^2
f_{ly}	Yield strength of longitudinal reinforcement, N/mm^2
K_c	$K_c = 0.3$ $F = \frac{l}{d} * V_f * d_f$
K	Coefficient dependent on the aspect ratio of the section
K_2	$K_2 = \left[0.2 * m + \sqrt{m} \left(\frac{0.45 * Y_1}{X_1} - \frac{S}{X_1 + Y_1} \right) \right.$
l	Length of fibres, mm
S	Stirrup spacing, mm
m	$m = \frac{\rho_l * f_{ly}}{\rho_t * f_{ty}}$
PT	Plastic theory
SBT	Skew bending theory
STA	Space truss analogy
t	Equivalent wall thickness, mm
X	Width of cross section, mm
Y	Height of cross section, mm
$X_\circ$	$X_o = \frac{5}{6} * X$
$Y_\circ$	$Y_o = Y - \frac{X}{6}$
X_1	Smaller centre to centre dimension of a closed rectangular stirrup, mm
Y_1	Larger centre to centre dimension of a closed rectangular stirrup, mm
V_f	Volume fraction of steel fibres, %
V_m	Volume of concrete without steel fibres, %
$V_\circ$	$V_o = \begin{cases} 0.005 & for \quad 0.85 * \rho_l \langle \rho_w \\ 0.002 & for \quad 0.85 * \rho_l \rangle \rho_w \end{cases}$
T_p	Torsional strength of plain concrete, N.mm
T_f	Torsional strength provided by steel fibres , N.mm

T_{pf}	Torsional strength provided by steel fibre reinforced concrete, N.mm
T_s	Torsional strength provided by transverse reinforcement, N.mm
T_u	Ultimate torsional strength of reinforced concrete contain steel fibres, N.mm
T_{UHPFRC}	Torsional strength contributed by UHPFRC, N.mm
T_{cr}	Cracking torsional strength, N.mm
ρ_L	Longitudinal reinforcement ratio, %
ρ_w	Web reinforcement ratio, %
η_o	Fibre orientation factor in elastic range and it is equal to 0.405
η_L	Ratio of average fibre stress to maximum fibre stress $\eta_L = \begin{cases} 0.5 & for \quad L_f \le L_{cr} \\ 1 - \dfrac{L_f}{2 * L_{cr}} & for \quad L_f \rangle L_{cr} \end{cases}$
α	Constant depends on the volume fraction of steel fibres and their value $\alpha = \begin{cases} 1.0 \; for \quad 0 \le V_f \le 0.5\% \\ 0.78 \; for \quad 0.5\% \le V_f \le 1.0\% \\ 0.65 \; for \quad Vf \ge 1.0\% \end{cases}$
α_p	$\alpha_p = 0.5 - \frac{X}{6 * Y}$
α_t	Variable depends on the dimension of the cross section $\alpha_t = 0.66 + 0.33 * \frac{Y_1}{X_1} \quad \le 1.5$
α_T	Slope of ultimate torque vs $\frac{X_1.Y_1.A_t.f_{ty}}{S}$
α_s	$\alpha_s = 0.5 - 0.223 * \frac{X}{Y}$
α_c	Coefficient for determining the torque supplied by the fibrous concrete
β	Constant of bond factor, in model (9),
β	$\beta = \begin{cases} 635278 * V_f^{\,2} - 450.83 * V_f + 2.56 \\ 7410 * V_f + 2.62 \quad for \quad (\end{cases}$
λ	Aspect ratio of steel fibre, $\lambda = \frac{l}{d}$
θ	Angle of inclination of spiral cracks, deg
ϕ	Angle of twist corresponds to an torque , rad
ϕ_u	Twisting angle at failure, rad $\phi_u \left(rad / m * 10^{-3} \right) = \begin{cases} 60 & for \quad 0.85 * \rho \\ 90 & for \quad 0.85 * \rho \end{cases}$
ϕ_{cr}	Twisting angle at the end of pre-cracking stage, rad
σ_{fc}	Shear stress in steel fibre reinforced concrete, N/mm^2
σ_c	Tensile strength of concrete, N/mm^2
τ_{fr}	Friction stress between fibre and matrix, N/mm^2
UHPFRC	Ultra-high performance fibre reinforced concrete
SFNSC	Steel fibre normal strength concrete

SFNHSC	Steel fibre normal and high strength concrete

Appendix B: Models

Authors	Model	No. of model
Elastic Theory		
Mansurand Paramasivam (1982)	$T_{pf} = K * X^2 * Y * f_t$	11
	$T_{pf} = T_p + T_f$	7
	$T_p = \frac{\pi * D^3}{16} * \sigma_{fc}$ for circular section	8
Tegas (1989)	$\sigma_{fc} = \left(1 - V_f\right) * \sigma_c + 2 * \beta * \tau * \frac{l}{d} * V_f$	9
	$T_f = \frac{1}{15}\left(\frac{l}{d}\right)^{3/2} * V_f * T_p$	10
Karayannis (2000)	$f_{ct} = \frac{2 * \eta_L * \eta_o * \tau_{fr} * V_f * L_f}{d_f}$	5
GunneswaraRao and Seshu (2003)	$T_{pf} = \alpha * X^2 * Y * f_t$	14
Fuad Okay and SerkanEngin(2012)	$T_{cr} = C_1 * h * b^2 * f_{rf}$	24
Plastic Theory		
Mansur and Paramasivam (1982)	$T_{pf} = \frac{X^2}{2} * \left(Y - \frac{X}{3}\right) * f_t$	12
GunneswaraRaoand Seshu (2003)	$T_{pf} = \alpha_p * X^2 * Y * f_t$	15
Skew Bending Theory		
Mansurand Paramasivam (1982)	$T_{pf} = 0.85 * \frac{X^2 * Y}{3} * f_r$	13
	$T_p = 0.13 * X^2 * Y * \sqrt{f_{c'}}$	19
Narayananand Kareem-Palanjian (1983)	$T_{cr} = T_p * \left[1 + K_c\left(\frac{l}{d} * V_f * d_f\right)\right]$	20
	$T_u = \left[0.326 - 0.035 * \frac{X}{Y}\right] * X^2 * Y *$	21
Craig *et al.*(1984)	$T_u = \alpha_T \cdot \frac{X_1.Y_1.A_t.f_y}{S} + \alpha_c.X^2.Y\sqrt{f_{c'}}$	34
Craig *et al.* (1986)	$T_{pf} = 1.56\left(X^2 + 10\right) * Y * \sqrt[3]{f_r^{\,2}}$	17
	$T_{pf} = 1.67\left(X^2 + 10\right) * Y * \sqrt[3]{f_{ct}^{\,2}}$	18
Wafa *et al.* (1992)	$T_{pf} = \frac{X^2 * Y}{3} * \alpha * f_{ctf}$	6
Chalioris and Karayannis (2009)	$T_{pf} = \frac{X^2 * Y}{3} * \left(0.71 * \quad f_{ctf}\right)$	1
	$T_{pf} = \frac{X^2 * Y}{3} * \left(0.71 * \quad f_{rf}\right)$	2
	$T_{pf} = 14.023\left(\frac{X^2}{64.516} + 10\right) * Y * \sqrt[3]{f_{cf}'}$	3
	$T_{pf} = \frac{X^2 * Y}{3}\left(0.85 * f_{rf}\right)$	4
Space truss analogy		
	$T_u = T_p + T_s + T_f$	30
Narayananand Kareem-Palanjian (1986)	$T_p = 0.13 * X^2 * Y * \sqrt{f_{c'}}$	31
	$T_f = 0.22 * \frac{X_o * Y_o}{X_o + Y_o} * X * Y * F * \sqrt{f_{c'}}$	32
	$T_s = K_2 * \frac{X_1 * Y_1}{S} * A_t * f_{ty}$	33
El-Niema (1993)	$T_u = T_p + T_s + T_f$	26
	$T_p = \frac{2.4}{\sqrt{X}} * X^2 * Y * \sqrt{f_c'}$	27
	$T_s = \alpha_t * \frac{X_1 * Y_1 * A_t * f_{ty}}{S}$	28
	$T_f = 0.22 * \frac{X_o * Y_o}{X_o + Y_o} * X * Y * F * \sqrt{f_{c'}}$	29
Joh *et al.* (2012)	$T_{cr} = 2 * f_{cr} * A_o * t$	35
	$T_u = T_s + T_{UHPFRC}$	36
	$T_s = \frac{2 * A_t * A_o * f_{ty}}{S} * \cot\theta$	37
	$T_{UHPFRC} = 2 * f_{rf} * t * A_o * \cot\theta$	38
Empirical and semi-empirical equations		
Narayananand Green(1980)	$f_t = \frac{f_{c'}}{15.5} + 0.80$	22
	$T_x = \frac{1}{2}\left(\frac{T_u}{\phi_u^{\,2}}\right) * \phi_x * \left(3 * \phi_u - \phi_x\right)$	23
GunneswaraRaoand Seshu (2003)	$T_{pf} = \alpha_s * X^2 * Y * f_t^{*}$	16
Fuad Okay and SerkanEngin (2012)	$T_u = T_{cr} + \beta * (\phi - \phi_{cr}) * \left(V_f - V_o\right) *$	25

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